年級:_____ 姓名:_____ 學號:_____ 考試範圍–常微分方程式

海洋大學河海工程學系2002 工程數學 (一) 二 B 班期中考解答 OPEN BOOK

1. If $g(x) = \frac{1}{2}(x + \frac{3}{x})$, determine $g(g(g(\cdots g(2)))) =?$ and $g(g(g(\cdots g(-2)))) =?$ (5 %)
If $y_{n+1}(x) = \int_0^x 2s\, y_n(s)ds + 1$, find $y_1(x), y_2(x)$ and $y_3(x)$ (3 %) determine $\lim_{n\to\infty} y_n(s) =?$ (7 %) where $y_0(x) = 1$.
Ans: $\sqrt{3}$ and $-\sqrt{3}$
Ans: e^{x^2}

2. Solve the Euler-Cauchy equation (20 %) $x^2y''(x) - 3xy'(x) + 4y(x) = 0$, using (1). $y_1(x) = x^m$, (2). Solve another complementary solution by using L'Hospital's rule, (3). Solve another complementary solution by using $y_2(x) = y_1(x)u_1(x)$. (4). Solve another complementary solution by using Wronskian.
Ans: $y_1 = x^2,\ y_2 = x^2 ln\, x$

3. Explain the excitation, beating and resonance mathematically and physically. (10 %)
Ans: 詳見講義
4. Given the following ordinary differential equations,and fill in the table (36 %)

$$\frac{dy}{dx} = \frac{2xy}{x^2 - y^2}, \tag{1}$$
$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}, \tag{2}$$
$$\frac{dy}{dx} = \frac{-2xy}{x^2 - y^2}, \tag{3}$$
$$\frac{dy}{dx} = \frac{x^2 - y^2}{2xy}, \tag{4}$$

ODE	Eq.(1)	Eq.(2)	Eq.(3)	Eq.(4)
Separable	No	No	No	No
Exact	No	No	Yes	Yes
Integrating factor	y^{-2}	x^{-2}	1	1
Homogeneous	Yes	Yes	Yes	Yes
Solution	$x^2 + y^2 = 2ky$	$x^2 + y^2 = 2kx$	$x^2y - \frac{y^3}{3} = c$	$\frac{x^3}{3} - xy^2 = c$
Orthogonality	Eq.(2)	Eq.(1)	Eq.(4)	Eq.(3)

5. Given the following first order ODEs,

$$y_1'(x) + y_1(x) = 2\, cos(x)$$
$$y_2'(x) + y_2(x) = 2\, sin(x)$$
$$y_3'(x) + y_3(x) = e^{-x}$$

solve their particular solutions :
(a). Rewrite $y_1(x) = A\, cos(x - \alpha)$, where $A > 0$, $0 < \alpha < 2\pi$, determine A and α. (5%)
(b). Rewrite $y_2(x) = B\, sin(x - \beta)$, where $B > 0$, $0 < \beta < 2\pi$, determine B and β. (5%)
(c). Solve $y_3(x)$. (10 %)
Ans:
(a). $y_1 = cos(x) + sin(x) = \sqrt{2}\, cos(x - 45^o)$, $A = \sqrt{2}$, $\alpha = \pi/4$.
(b). $y_2 = sin(x) - cos(x) = \sqrt{2}\, sin(x - 45^o)$, $B = \sqrt{2}$, $\beta = \pi/4$.
(c). $y_3(x) = xe^{-x}$.