國立臺灣海洋大學河海工程學系 2002 工程數學 (一) 第三次小考解答

1. Solve $y' = y(1-y)$, $y(0) = 0.5$ using separable form.

$$y' = y(1-y) \Rightarrow \frac{dy}{y(1-y)} = dx$$

$$\int \frac{dy}{y(1-y)} = \int dx \Rightarrow \int (\frac{1}{y} + \frac{1}{1-y})dy = \int dx$$

$$ln|y| - ln|1-y| = x + c \Rightarrow ln|\frac{y}{1-y}| = x + c$$

$$|\frac{y}{1-y}| = ke^x$$

$$y(0) = 0.5 \Rightarrow k = 1$$

$$\frac{y}{1-y} = e^x \Rightarrow y = \frac{e^x}{1+e^x}$$

2. Solve $y' - y = -y^2$, $y(0) = 0.5$ using Bernoulli form.

$$y' - y = -y^2, u = y^{-1} \Rightarrow du = -\frac{1}{y^2}dy$$

$$u' + u = 1 \Rightarrow u' = 1 - u$$

$$\frac{1}{1-u}du = dx \Rightarrow \int \frac{1}{1-u}du = \int dx$$

$$-ln|1-u| = x + c \Rightarrow 1 - u = ke^{-x}$$

$$1 - \frac{1}{y} = ke^{-x}$$

$$y(0) = 0.5 \Rightarrow k = -1$$

$$1 - \frac{1}{y} = -e^{-x}$$

$$\frac{1}{y} = 1 + e^{-x} \Rightarrow y = \frac{1}{1+e^{-x}} = \frac{e^x}{1+e^x}$$

Comments: The results of two methods match well.