

1. Given the nonconstant coefficient second order ODE

$$x^2y''(x) - 2xy' - 10y = -10$$

(1). Assume the $y = x^n$ for the complementary solution, determine n .

$$n^2 - 3n - 10 = 0$$

$$n = 5 \text{ or } -2$$

(2). If $y_1(x) = 1/x^2$ is one of the complementary solution, please determine the other one $y_2(x)$ by method of variations of parameters, $y_2(x) = y_1(x)u_1(x)$. Please find $u_1(x)$.

$$u_1(x) = \frac{c_1}{7}x^7 + c_2$$

(3). Solve the particular solution by $y_p(x) = y_1(x)v_1(x) + y_2(x)v_2(x)$, where

$$y_1v'_1 + y_2v'_2 = 0$$

$$y'_1v'_1 + y'_2v'_2 = -10/x^2$$

Please determine v_1, v_2 and y_p .

$$v_1 = \frac{5}{7}x^2 + c_1$$

$$v_2 = \frac{2}{7}x^{-5} + c_2$$

$$y_p = 1 + \frac{c_1}{x^2} + c_2x^5$$

(4). By changing variable, $x = e^t$ and $y(x) = y(e^t) = Y(t)$, then determine the ODE for $Y(t)$ and solve $Y(t)$ and $y(x)$.

$$Y''(t) - 3Y'(t) - 10Y(t) = 0$$

$$\text{Let } Y(t) = e^{mt}$$

$$m^2 - 3m - 10 = 0$$

$$m = 5 \text{ or } -2$$

$$Y(t) = e^{5t} \text{ or } Y(t) = e^{-2t}$$

$$y(x) = x^5 \text{ or } y(x) = x^{-2}$$