

1. Given the nonconstant coefficient second order ODE

$$x^2y''(x) - 2xy' - 10y = -10$$

(1). Assume the $y = x^n$ for the complementary solution, determine n.

(2). If $y_1(x) = 1/x^2$ is one of the complementary solution, please determine the other one $y_2(x)$ by method of variations of parameters, $y_2(x) = y_1(x)u_1(x)$. Please find $u_1(x)$.

(3). Solve the particular solution by $y_p(x) = y_1(x)v_1(x) + y_2(x)v_2(x)$, where

- $y_1v_1' + y_2v_2' = 0$
- $y_1'v_1' + y_2'v_2' = -10/x^2$

Please determine v_1, v_2 and y_p .

(4). By changing variable, $x = e^t$ and $y(x) = y(e^t) = Y(t)$, then determine the ODE for Y(t) and solve Y(t) and y(x).