國立臺灣海洋大學河海工程學系 2003 工程數學 (一) 期末考 (closed book)

1 Given an ODE

$$4x\,y''(x)+2\,y'(x)+y(x)=0$$

Is $x=0$ singular point ? (5 %) Is $x=0$ regularly singular point ? (5 %) By changing the variable of $x=t^2$, transform the original ODE to the new ODE for $Y(t)$ and solve $Y(t)$ and $y(x)$, where $y(x)=Y(t)$. (10 %) By setting $y=\sum_{n=0}^{\infty}c_n x^{n+r}$, determine the indicial equation. (5 %) Please also determine the indicial equation of $-x^2y''(x)-2xy'(x)+56y(x)=0$. (5 %)

2 Fill in the table [A], [B], [C] ... [J] (20 %)

Sturm-Liouvilli System $(py')'+qy=-\lambda\rho y$

Function	ODE	$p(x)$	$q(x)$	$\rho(x)$	λ
Legendre polynomial	$(1-x^2)y''-2xy'+N(N+1)y=0$	$[A]$	0	1	$[B]$
Bessel function	$x^2y''+xy'+(x^2-\nu^2)y=0$	$[C]$	x	$[D]$	$-\nu^2$
Simple Harmonic Motion function	$[E]$	1	0	1	ω^2
Hermite function	$y''-2xy'+2\alpha y=0$	$[F]$	0	e^{-x^2}	2α
Laguerre function	$xy''+(1-x)y'+ny=0$	$[G]$	0	$[H]$	n
Chebyshev polynomial	$(1-x^2)y''-xy'+n^2y=0$	$[I]$	0	$[J]$	n^2

Write down the orthogonal properties of Legendre and Chebyshev polynomials. (10 %)

3 Find the Laplace transform of 1, t, $cos(t)$, $sin(t)$, e^t, e^{-t}, $cosh(t)$, $sinh(t)$. (20 %)

4 We define the following functions,

$$u(t)=1, t>0;\ \text{ otherwise }\ u(t)=0$$

$$p(t)=1, t>1;\ \text{ otherwise }\ p(t)=0$$

$$q(t)=1, 0<t<1;\ \text{ otherwise }\ q(t)=0$$

Please find the Laplace transform of $u(t), p(t)$ and $q(t)$. (15 %) What is the definition of convolution ? Write down the mathematical formula and its geometric meaning. (10 %) Please find the convolution of $q(t)$ and $q(t)$. (10 %)

5 If we define the correlation function of $a(t)$ and $b(t)$ by

$$c(t)=\int_{-\infty}^{\infty}a(\tau)b(t+\tau)d\tau$$

Please explain its geometric meaning (5 %) and determine the correlation of $q(t)$ and $q(t)$. (10%)