

1 Given an ODE

$$4x y''(x) + 2 y'(x) + y(x) = 0$$

Is  $x = 0$  singular point? (5 %) Is  $x = 0$  regularly singular point? (5 %) By changing the variable of  $x = t^2$ , transform the original ODE to the new ODE for  $Y(t)$  and solve  $Y(t)$  and  $y(x)$ , where  $y(x) = Y(t)$ . (10 %) By setting  $y = \sum_{n=0}^{\infty} c_n x^{n+r}$ , determine the indicial equation. (5 %) Please also determine the indicial equation of  $-x^2 y''(x) - 2xy'(x) + 56y(x) = 0$ . (5 %)

Ans: Yes; Yes;  $Y''(t) + Y(t) = 0$ ;  $Y(t) = C_1 \cos t + C_2 \sin t$ ;  $Y(x) = C_1 \cos \sqrt{x} + C_2 \sin \sqrt{x}$ ;  $r^2 - \frac{1}{2}r = 0$ ;  $r^2 + r - 56 = 0$

2 Fill in the table [A], [B], [C] ... [J] (20 %)

Sturm-Liouville System $(py')' + qy = -\lambda \rho y$
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Function	ODE	$p(x)$	$q(x)$	$\rho(x)$	$\lambda$
Legendre polynomial	$(1 - x^2)y'' - 2xy' + N(N + 1)y = 0$	$(1 - x^2)$	0	1	$N(N + 1)$
Bessel function	$x^2 y'' + xy' + (x^2 - \nu^2)y = 0$	$x$	$x$	$\frac{1}{x}$	$-\nu^2$
Simple Harmonic Motion function	$y'' + w^2 y = 0$	1	0	1	$\omega^2$
Hermite function	$y'' - 2xy' + 2\alpha y = 0$	$e^{-x^2}$	0	$e^{-x^2}$	$2\alpha$
Laguerre function	$xy'' + (1 - x)y' + ny = 0$	$xe^{-x}$	0	$e^{-x}$	$n$
Chebyshev polynomial	$(1 - x^2)y'' - xy' + n^2 y = 0$	$\frac{1}{\sqrt{1-x^2}}$	0	$\frac{1}{\sqrt{1-x^2}}$	$n^2$

Write down the orthogonal properties of Legendre and Chebyshev polynomials. (10 %)

Ans:  $\int_{-1}^1 P_n(x)P_m(x)dx = 0$ ;  $\int_{-1}^1 C_n(x)C_m(x)\frac{1}{\sqrt{1-x^2}}dx = 0$

3 Find the Laplace transform of 1,  $t$ ,  $\cos(t)$ ,  $\sin(t)$ ,  $e^t$ ,  $e^{-t}$ ,  $\cosh(t)$ ,  $\sinh(t)$ . (20 %)

Ans:  $\frac{1}{s}$ ;  $\frac{1}{s^2}$ ;  $\frac{s}{s^2+1}$ ;  $\frac{1}{s^2+1}$ ;  $\frac{1}{s-1}$ ;  $\frac{1}{s+1}$ ;  $\frac{s}{s^2-1}$ ;  $\frac{1}{s^2-1}$

4 We define the following functions,

$$u(t) = 1, t > 0; \text{ otherwise } u(t) = 0$$

$$p(t) = 1, t > 1; \text{ otherwise } p(t) = 0$$

$$q(t) = 1, 0 < t < 1; \text{ otherwise } q(t) = 0$$

Please find the Laplace transform of  $u(t)$ ,  $p(t)$  and  $q(t)$ . (15 %) What is the definition of convolution ? Write down the mathematical formula and its geometric meaning. (10 %) Please find the convolution of  $q(t)$  and  $q(t)$ . (10 %)

Ans:  $\frac{1}{s}; \frac{e^{-s}}{s}; \frac{1}{s}(1 - e^{-s})$ ; 摺, 右平移  $t$ , 乘積, 積分( $0, t$ );  $f(t) * g(t) = \int_0^t f(\tau)g(t - \tau)d\tau$ ;  
 $\begin{cases} t, 0 < t < 1 \\ 2 - t, 1 < t < 2 \end{cases}$  5 If we define the correlation function of  $a(t)$  and  $b(t)$  by

$$c(t) = \int_{-\infty}^{\infty} a(\tau)b(t + \tau)d\tau$$

Please explain its geometric meaning (5 %) and determine the correlation of  $q(t)$  and  $q(t)$ . (10%)

Ans: 不摺, 左平移  $t$ , 乘積, 積分( $-\infty, \infty$ );  $\begin{cases} 1 - t, & 0 < t < 1 \\ 1 + t, & -1 < t < 0 \end{cases}$

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