班級：_____學號：_____姓名：_____

國立台灣海洋大學河海工程學系 2002 工程數學第二次作業解答

1. Given $y' = \frac{1}{x}y^2 + \frac{1}{x}y - \frac{2}{x}$, we solved $y(x)$ after the known solution of $y_0(x)=1$ in the course. Please try again by choosing $y_0(x)=-2$

Sol:

$$y = -2 + \frac{1}{z}$$

$$\frac{dy}{dx} = -\frac{1}{z^2}\frac{dz}{dx}$$

$$-\frac{1}{z^2}\frac{dz}{dx} = \frac{1}{x}\left(-2+\frac{1}{z}\right)^2 + \frac{1}{x}\left(-2+\frac{1}{z}\right) - \frac{2}{x}$$

$$-\frac{1}{z^2}\frac{dz}{dx} = \frac{-3}{xz} + \frac{1}{xz^2} \quad \Rightarrow \quad \frac{dz}{dx} = \frac{3z}{x} - \frac{1}{x}$$

$$\frac{dz}{dx} - \frac{3}{x}z = -\frac{1}{x} \quad \Rightarrow \quad x^3z' - 3x^2z = -x^2$$

$$\left(x^3z\right)' = -x^2 \quad \Rightarrow \quad x^3z = -\frac{1}{3}x^3 + c \quad \Rightarrow \quad z = -\frac{1}{3} + \frac{c}{x^3}$$

$$\because y = 1 + \frac{1}{z} \qquad \therefore y = 1 + \left(\frac{1}{-\frac{1}{3} + \frac{c}{x^3}}\right)$$