班級：_____學號：_____姓名：_____

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$y''-\frac{4}{x}y'-\frac{6}{x^2}y=-\frac{6}{x^2}$,$y_p=y_1u_1+y_2u_2$，找出 u_1,u_2 使得 $y_p=1$

Sol: (1) $y''-\frac{4}{x}y'-\frac{6}{x^2}y=0$

$$y=x^m \qquad y'=mx^{m-1} \qquad y''=m(m-1)x^{m-2}$$

$$y=x^m \qquad x\,y'=m\,x^m \qquad x^2y''=m(m-1)x^m$$

$$m(m-1)x^m-4mx^m-6x^m=0 \quad \Rightarrow \quad m=-1 \ \textbf{or} \ m=6$$

$$y_1=\frac{1}{x} \qquad y_2=x^6$$

(2) $y_P''-\frac{4}{x}y_P'-\frac{6}{x^2}y_P=-\frac{6}{x^2}$

$$y_P=y_1u_1+y_2u_2$$
$$y_P'=y_1'u_1+y_1u_1'+y_2u_2'+y_2'u_2$$
$$y_P''=y_1''u_1+y_1'u_1'+y_2''u_2+y_2'u_2'$$

$$[y_1''u_1+y_1'u_1'+y_2''u_2+y_2'u_2']-\frac{4}{x}[y_1'u_1+y_2'u_2]-\frac{6}{x^2}[y_1u_1+y_2u_2]=-\frac{6}{x^2}$$

$$y_1'u_1'+y_2'u_2'=-\frac{6}{x^2}$$

將 y_1 **與** y_2 **代入 ：**

$$\begin{cases}\frac{1}{x}u_1'+x^6u_2'=0\\ \left(\frac{1}{x}\right)'u_1'+\left(x^6\right)'u_2'=-\frac{6}{x^2}\end{cases} \quad \Rightarrow \quad u_1'=\frac{6}{7} \ \textbf{與} \ u_2'=-\frac{6}{7x^7}$$

解得： $u_1=\frac{6}{7}x+c_1$ **與** $u_2=\frac{1}{7x^6}+c_2$

$$y_P=y_1u_1+y_2u_2=1+\frac{c_1}{x}+c_2x^6$$