班級：_____學號：_____姓名：_____

國立台灣海洋大學河海工程學系 2002 工程數學第四次作業解答

Solve $y''-2y'+y=0$

(1)Use L'Hopital's rule

(2)Use $y_2=y_1u_1$

(3)Use Wronskian method

Sol:

(1) Let $y=e^{st}$

$$(s^2-2s+1)=0 \quad \Rightarrow \quad s=1,1$$

$$\lim_{s\to 1} e^{st}=e^t$$

$$\lim_{s\to 1}\frac{e^{st}-e^t}{s-1}=te^t$$

(2) Let $y=e^{st}$

$$(s^2-2s+1)=0 \quad \Rightarrow \quad s=1,1$$

$$y_2=y_1u_1=u_1e^t$$

$$y_2'=u_1e^t+u_1'e^t$$

$$y_2''=u_1e^t+2u_1'e^t+u_1''e^t$$

$$(u_1e^t+2u_1'e^t+u_1''e^t)-2(u_1e^t+u_1'e^t)+u_1e^t=0$$

$$u_1''=0 \quad \Rightarrow \quad u_1=at+b$$

$y_2=(at+b)e^t$ **where a,b are constants**

(3) Let $y=e^{st}$

$$(s^2-2s+1)=0 \quad \Rightarrow \quad s=1,1$$

$$\begin{vmatrix} e^t & y_2 \\ e^t & y_2' \end{vmatrix}=e^t(y_2'-y_2)=e^{-\int -2dt}=c_1e^{2t}$$

$$y_2'-y_2=c_1e^t$$

$$e^{-t}y_2'-e^{-t}y_2=c_1$$

$$(e^{-t}y_2)'=c_1$$

$$e^{-t}y_2=c_1t+c_2$$

$y_2=(c_1t+c_2)e^t$ **where** c_1, c_2 **are constants**