海洋大學河海工程學系2002 二 B 工程數學 (一) 第七次作業解答

1. Given the following ODE,

$$t^2\ddot{y}(t) - 4t\dot{y}(t) + 6y(t) = 0$$

Solve the ODE by using Laplace transform. Taking Laplace transform again, what will happen ?

$$\begin{aligned}
t \text{ domain} &\rightarrow s \text{ domain} && (1)\\
y(t) &\rightarrow Y(s) && (2)\\
\dot{y}(t) &\rightarrow sY(s) - y(0) && (3)\\
t\dot{y}(t) &\rightarrow -sY'(s) - Y(s) && (4)\\
\ddot{y}(t) &\rightarrow s^2Y(s) - y(0)s - \dot{y}(0) && (5)\\
t\ddot{y}(t) &\rightarrow -2sY(s) - s^2Y'(s) + y(0) && (6)\\
t^2\ddot{y}(t) &\rightarrow 4sY'(s) + s^2Y''(s) + 2Y(s) && (7)\\
t^2\ddot{y}(t) - 4t\dot{y}(t) + 6y(t) = 0 &\rightarrow s^2\ddot{Y}(s) + 8s\dot{Y}(s) + 12Y(s) = 0 && (8)
\end{aligned}$$

$$at^2\ddot{y}(t) + bt\dot{y}(t) + cy(t) = 0$$

$$as^2\ddot{Y}(s) + (4a-b)s\dot{Y}(s) + (2a-b+c)Y(s) = 0$$

$$au^2\ddot{Z}(u) + (4a-(4a-b))u\dot{Z}(u) + (2a-(4a-b)+2a-b+c)Z(u) = 0$$

It can be reduced to

$$au^2\ddot{Z}(u) + bu\dot{Z}(u) + cZ(u) = 0$$

$$y(t) \rightarrow Y(s) \rightarrow Z(u)$$