(B)   (ODE)

1. In the course, we solved the Ricatti equation $y' = \frac{1}{x}y^2 + \frac{1}{x}y - \frac{2}{x}$

   by using the solution $y_2 = y_1 + \frac{1}{z}$ with $y_1 = 1$,

   we obtain $y_2 = \dfrac{1}{-\dfrac{1}{3} + \dfrac{c}{x^3}} + 1$, $c \in R$

   By setting $c = 0$, we have $y_2 = -2$, solve $y_3 = -2 + \frac{1}{z}$,

   please find $y_3 = ?$ (20%)

2. $\dfrac{dy}{dx} = \dfrac{2xy}{x^2 - y^2}$

(1) Separable (Yes or No)? Exact (Yes or No)? Homogeneous (Yes or No)   (15%)
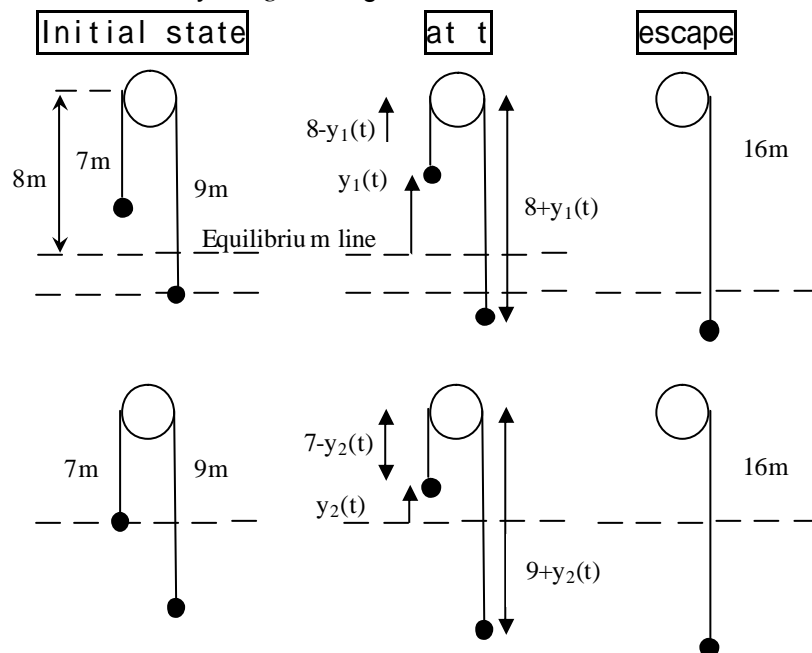(2) Find the integrating factor. (5%)
(3) Find $z_1(x, y) = c$ and plot contour. (5%)
(4) If $\dfrac{dy}{dx} = \dfrac{y^2 - x^2}{2xy}$, find $z_2(x, y) = c$ and plot contour. (5%)
(5) Please compare figures in (4) and (5), and explain. (5%)

3. Given $\dfrac{dy}{dx} + \dfrac{2}{x}y = x^2$, solve $y(x)$ (10%)

4. Cable density=$1 kg/m$, g=$9.8 m/\sec^2$

Initial state   at t   escape

$16y_1'' = 2y_1 \cdot 9.81$

$y_1(0) = 1$

$y_1'(0) = 0$

$16y_2'' = ?$ (5%)

$y_2(0) = ?$ (5%)

$y_2'(0) = 0$

(1) When does it escape the roller? (10%) Can you determine by energy conservation? (5%)
(2) What is the escaping velocity? (10%) Can you determine by energy conservation? (5%)
5. Solve the Clairauts equation $y = xy' + y'^2$, (10%) and plot. (5%)