I. Characteristic curves for $f(s)$ and $g(s)$:

$ag'^2(s) - 2bf'g' + cf'^2 = 0$

Compared with

$ay^2 - 2byx + cx^2 = 0$

reduce to

$a\kappa^2 - 2b\kappa + c = 0$

where

$\kappa = \dfrac{g'(s)}{f'(s)}$

II. Elliptic

$b^2 - 4ac = 0$

III. Parabolic

$b^2 - 4ac < 0$

VI. Hyperbolic

$b^2 - 4ac > 0$