Quasi-linear first order equation

I. $u(x, y)$:
II. Quasi-linear PDE:

$$a(x, y, u)u_x + b(x, y, u)u_y = c(x, y, u)$$

III. Linear PDE:

$$a(x, y)u_x + b(x, y)u_y = 0$$

IV. Example: Burger's equation

$$u_x + uu_y = 0$$

IV. Assume $z = u(x, y)$

$$z = constant$$

is the solution of

$$(u_x, u_y, -1) \cdot (a, b, c) = 0$$

V. A curve in parametric form

$$\frac{dx}{dt} = a$$

$$\frac{dy}{dt} = b$$

$$\frac{du}{dt} = c$$

with the initial conditions

$$x(0, s) = p(s)$$

$$y(0, s) = q(s)$$

$$z(0, s) = r(s)$$

The solution of the surface in parametric form is

$$x(t, s)$$

$$y(t, s)$$

$$z(t, s)$$