number of methods available for solving differential equations which are entirely algebraic in nature, which usually reduce differential equation computations to a computation of integrals. However, unlike the case of the indefinite integrations, there is no known complete decision procedure for solving differential equations. For differential equations where a closed form solution cannot be determined, often one can compute an approximate solution based on numerical schemes. PD Ease, a computer software package offered by MACSYMA Inc., is one such example.

1.2 Comparisons of Symbolic and Numerical Computational Methodologies

1.2.1 History and Preliminary Comparisons of Different Packages

Table 1.2.1-1 Some important milestones in computer software development related to this course.

Year	Milestone	Creators/Leaders	Organizations
1958	FORTRAN[1] [FORmula TRANslator]	John Backus	IBM Corp.
1960	LISP[2] [LISt Processing]	John McCarthy	MIT
1961	SAINT[3] [Symbolic Automation INTegration]	James Slagle	MIT
1965	MATHLAB[4]	Carl Engelman	MIT
1968	REDUCE[5]	Tony Hearn	Stanford University
1971	MACSYMA[6] (LISP)	Joel Moses, William Martin, et al.	MIT
1972	C[7]	Dennis Ritchie	AT&T Bell Labs.
~1975	muMATH[8]	David Stoutemyer, Albert Rich	University of Hawaii
1981	MAPLE[9] (C)	Gaston Gonnet, Keith Geddes	University of Waterloo
~1987	DERIVE[10]	David Stoutemyer, Albert Rich	University of Hawaii
~1988	AXIOM[11]	Richard Jenks, Barry Trager, Stephen Watt	IBM Thomas J. Watson Res. Center
1988	MATHEMATICA[12] (C)	Stephen Wolfram	SMP at Caltech

Note:

1. FORTRAN is the most popular scientific numerical computation language.
2. LISP is a major advancement in computer language for symbolic computation. LISP is integer based instead of floating point based such as FORTRAN. Even though operations such as symbolic differentiation which is foreign to languages like FORTRAN, it is relatively easy for LISP.
3. SAINT is one of the earliest applications of LISP to symbolic computation.

4. MATHLAB is a LISP based system for the manipulation of polynomials and rational functions. It is also the first interactive system designed to be used as a symbolic calculator.
5. REDUCE was first developed as an interactive LISP based system for physics calculations and then redesigned to become REDUCE 2 (1970) as a general purpose system with special facilities for high-energy physics calculations.
6. MACSYMA was the most ambitious symbolic computation package of the 1970s. It included facilities to aid in computational areas such as limit calculations, solution of equations and symbolic integrations. Many of the standard techniques (e.g. integration of elementary functions, Hensel lifting, space modular algorithms) in use today either came from, or were strongly influenced by the research group at MIT. It is by far the most powerful existing computer algebra system.
7. C is a mid-level language that offers programmer many levels of freedom controlling the system resources. Many of the latter symbolic computational packages were developed using C.
8. The muMATH was written in a small subset of LISP. It was the first comprehensive computer algebra system which could actually run on the IBM family of PC computers.
9. MAPLE was designed with a modular structure, which consists of a small compiled kernel of modest powers, implemented completely in the system implementation language C and a large mathematical library of routines written in the user level MAPLE language to be interpreted by the kernel. The small size kernel allowed MAPLE to be implemented in a number of smaller platforms and allowed multiple users to access it on time-sharing systems. Its large mathematical library, on the other hand, allowed it to be powerful enough to meet the mathematical requirements of researchers.
10. DERIVE has an impressive range of applications considering the limitations of the 16-bit PC machines for which it was designed. Since it was designed to be used as an interactive system and not as a programming environment, DERIVE has a friendly interface, with such added features as two-dimensional input editing of mathematical expressions and 3-D plotting facilities.
11. AXIOM was originally known as SCRATCHPAD II. It is the only "strongly typed" computer algebra system. Although other computer algebra systems developed algorithms for a specific collection of algebraic domains (e.g. the domain of polynomials over the integers or the field of rational numbers), AXIOM allows users to write algorithms over general fields or domains.
12. MATHEMATICA was the first to integrate symbolics, numerics and graphics in a single software package. It is one of the most powerful packages available on an IBM compatible PC platform at the time of its introduction. It is also one of the first systems to successfully illustrate the advantages of combining a computer algebra system with easy to use editing features on machines designed for graphical user-interface such as MS-Windows. The programming language of MATHEMATICA closely follows the rule-based approach of its predecessor SMP (Symbolic Manipulation Program).

Reference:

1. "Proceedings of the Second Symposium on Symbolic and Algebraic Manipulation (SYMSAM '71)," Los Angeles, ed. by S. R. Petrick, ACM Press, New York (1971)
2. "Proceedings of the 1968 Summer Institute on Symbolic Mathematical Computation (ed. by R. G. Tobey)," IBM Programming Lab. Rep. FSC69-0312 (1969).
3. K. O. Geddes, S. R. Czapor, and G. Labahn, "Algorithms for Computer Algebra," Kluwer Academic Publishers, London (1992).

1.2.1 Benefit/Drawback

	Symbolic Computations	Numerical Computations
Possesses Infinite Precision	**	
Derives Equations/Model	**	
Approximates Difficult Physical Problems Efficiently		**
Evaluates Mathematical Expressions by Numbers		**
Others	??	??

It is natural and beneficial to integrate symbolic computations and numerical computations so as to get the full benefit brought to us with the development of both numerical and symbolic computational packages available today. This understanding is also one of the underlying reasons for many of the instructors at National Taiwan University to join forces in offering this course.

1.3 Examples of Some Famous Software Packages in a MS-Windows Environment

1.3.1 MATHEMATICA

Some people have trouble getting started with Mathematica because of small, minor details. Some of the important special forms are listed in Appendix 1.6.5 (A table of specific forms for Mathematica) for your reference in order to facilitate your first experience with Mathematica.

1.3.1.1 Introduction

Units can be used within calculations.

```
In[1]:=
27 meters + 5 meters
Out[1]:
32 meters
```

Permutations can be performed within Mathematica as well.

```
In[1]:=
Permutations[{Mary, Adam, Howard}]
```