邊界元素法期中考

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1. Explain the following items. (50%)
   (a). dual integral formulation and dual boundary element method
   (c). hypersingular integral equation and Hadamard principal value
   (d). kernel function and order analysis
   (e). singular integral equation and Cauchy principal value
   (f). difference type kernel and two point function
   (g). interior and exterior problem
   (h). Green's function, fundamental solution and free space Green's function
2. Why dual integral formulation ? (10%)
3. The fundamental solution is defined as follows

$$\frac{d^2U(x,s)}{dx^2} = \delta(x-s)$$

The dual integral equations are shown

$$u(x) = [T(s,x)u(s) - U(s,x)\frac{du(s)}{ds}] \mid_{s=0}^{s=1}$$

$$\frac{du(x)}{dx} = [M(s,x)u(s) - L(s,x)\frac{du(s)}{ds}] \mid_{s=0}^{s=1}$$

   (a). Determine $U(s,x), T(s,x), L(s,x)$ and $M(s,x)$ for $x > s$ and $x < s$. (10%)
   (b). Plot $U(s,x), T(s,x), L(s,x)$ and $M(s,x)$ versus x for $0 < x, s < 1$. (10%)
   (c). Determine (10%)

$$\lim_{x\to 1^-} U(0,x) =?, \lim_{x\to 0^+} T(0,x) =?, \lim_{x\to 1^-} L(0,x) =?, \lim_{x\to 0^+} M(1,x) =?$$

   (d). Based on the dual integral formulation, solve (10%)

$$\frac{d^2u(x)}{dx^2} = 0$$

subject to $u(0) = 0, u(1) = 1$. Any comments on the L, M equation ?

4. After extending 2-D Laplace to 3-D Laplace equation, we have

$$U(s,x) = 1/r$$

Find the explicit form for $T(s,x), L(s,x)$ and $M(s,x)$ (10%) and prove (10%)

$$T(s,x) = L(x,s)$$

$$M(s,x) = M(x,s)$$