# 邊界元素法期中考 by J. T. Chen

考試時間 — 18:00 to 21:00, April,24,1996 考試方式 — OPEN BOOK

1. The fundamental solution $U_1(x,s)$ satisfies (10%)
$$\frac{d^2U_1(x,s)}{dx^2} = \delta(x-s) \tag{1}$$
where
$$U_1(x,s) = \begin{cases} \frac{1}{2}(x-s), & x > s, \\ -\frac{1}{2}(x-s), & x < s \end{cases} \tag{2}$$
the boundary integral equation can be obtained as
$$u(s) = \frac{\partial U_1(x,s)}{\partial x}u(x)\Big|_0^1 - U_1(x,s)\frac{du(x)}{dx}\Big|_0^1 \tag{3}$$
Please derive the stiiffness matrix of $K$ such that
$$K\mathbf{u} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}\begin{bmatrix} u(0) \\ u(1) \end{bmatrix} = \begin{bmatrix} \frac{du(x)}{dx}\big|_{x=0} \\ \frac{du(x)}{dx}\big|_{x=1} \end{bmatrix}$$
2. If $U_2(x,s) = 2\pi U_1(x,s), i.e.,$ (10%)
$$U_2(x,s) = \begin{cases} \pi(x-s), & x > s, \\ -\pi(x-s), & x < s \end{cases} \tag{4}$$
the boundary integral equation can be obtained
$$\alpha u(s) = \frac{\partial U_2(x,s)}{\partial x}u(x)\Big|_0^1 - U_2(x,s)\frac{du(x)}{dx}\Big|_0^1 \tag{5}$$
Please determine the value of $\alpha$=?
Please derive the stiiffness matrix of $K$ such that
$$K\mathbf{u} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}\begin{bmatrix} u(0) \\ u(1) \end{bmatrix} = \begin{bmatrix} \frac{du(x)}{dx}\big|_{x=0} \\ \frac{du(x)}{dx}\big|_{x=1} \end{bmatrix}$$
and compare with Problem 1.
3. If $U_3(x,s) = U_1(x,s) + b$ where $b$ is a constant, we can have (10%)
$$u(s) = \frac{\partial U_3(x,s)}{\partial x}u(x)\Big|_0^1 - U_3(x,s)\frac{du(x)}{dx}\Big|_0^1 \tag{6}$$
Please derive the stiiffness matrix of $K$ such that
$$K\mathbf{u} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}\begin{bmatrix} u(0) \\ u(1) \end{bmatrix} = \begin{bmatrix} \frac{du(x)}{dx}\big|_{x=0} \\ \frac{du(x)}{dx}\big|_{x=1} \end{bmatrix}$$
and compare with Problem 1 and 2.

4. If $U_4(x,s) = U_1(x,s) + ax + b$ where $a$ and $b$ are constants, we can have (10%)

$$u(s) = \frac{\partial U_4(x,s)}{\partial x} u(x)\Big|_0^1 - U_4(x,s)\frac{du(x)}{dx}\Big|_0^1 \tag{7}$$

Please derive the stiiffness matrix of $K$ such that

$$K\mathbf{u} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} u(0) \\ u(1) \end{bmatrix} = \begin{bmatrix} \frac{du(x)}{dx}|_{x=0} \\ \frac{du(x)}{dx}|_{x=1} \end{bmatrix}$$

and compare with Problem 1, 2 and 3.

5. Explain the following items. (30%)
   (a). dual integral formulation and dual boundary element method
   (b). superstrong integral equation and Mangler's principal value
   (c). hypersingular integral equation and Hadamard principal value
   (c). singular integral equation and Cauchy principal value
   (d). kernel function and order analysis
   (f). difference type kernel and two point function
   (g). interior and exterior problem
   (h). fundamental solution and free space Green's function
   (i). Green's function, moment diagram and influence line
   (j). divergent integral and divergent series

6. In the course, we have $U(s,x) = ln(r)$ for 2-D Laplace equation, please extend to 3-D Laplace equation, such that

$$\nabla^2 U(s,x) = \delta(x-s)$$

Find $U(x,s)$ by one method you can. (10%)
Find the explicit form for $T(s,x), L(s,x)$ and $M(s,x)$ (10%) and prove (10%)

$$U(s,x) = U(x,s)$$

$$T(s,x) = L(x,s)$$

$$M(s,x) = M(x,s)$$

7. What are the roles for hypersingularity in BEM ? (more than five roles) (10%)