邊界元素法第三次作業 by J. T. Chen

1. The fundamental solution $U_1(x,s)$ satisfies (10%)

$$\frac{d^2U_1(x,s)}{dx^2} = \delta(x-s) \tag{1}$$

where

$$U_1(x,s) = \begin{cases} \frac{1}{2}(x-s), & x > s, \\ -\frac{1}{2}(x-s), & x < s \end{cases} \tag{2}$$

the boundary integral equation can be obtained as

$$u(s) = \frac{\partial U_1(x,s)}{\partial x}u(x)\bigg|_0^1 - U_1(x,s)\frac{du(x)}{dx}\bigg|_0^1 \tag{3}$$

Please derive the stiiffness matrix of K such that

$$K\mathbf{u} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}\begin{bmatrix} u(0) \\ u(1) \end{bmatrix} = \begin{bmatrix} \frac{du(x)}{dx}\big|_{x=0} \\ \frac{du(x)}{dx}\big|_{x=1} \end{bmatrix}$$

2. If $U_2(x,s) = 2\pi U_1(x,s), i.e.,$ (10%)

$$U_2(x,s) = \begin{cases} \pi(x-s), & x > s, \\ -\pi(x-s), & x < s \end{cases} \tag{4}$$

the boundary integral equation can be obtained

$$\alpha u(s) = \frac{\partial U_2(x,s)}{\partial x}u(x)\bigg|_0^1 - U_2(x,s)\frac{du(x)}{dx}\bigg|_0^1 \tag{5}$$

Please determine the value of α=?

Please derive the stiiffness matrix of K such that

$$K\mathbf{u} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}\begin{bmatrix} u(0) \\ u(1) \end{bmatrix} = \begin{bmatrix} \frac{du(x)}{dx}\big|_{x=0} \\ \frac{du(x)}{dx}\big|_{x=1} \end{bmatrix}$$

and compare with Problem 1.

3. If $U_3(x,s) = U_1(x,s) + b$ where b is a constant, we can have (10%)

$$u(s) = \frac{\partial U_3(x,s)}{\partial x}u(x)\bigg|_0^1 - U_3(x,s)\frac{du(x)}{dx}\bigg|_0^1 \tag{6}$$

Please derive the stiiffness matrix of K such that

$$K\mathbf{u} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}\begin{bmatrix} u(0) \\ u(1) \end{bmatrix} = \begin{bmatrix} \frac{du(x)}{dx}\big|_{x=0} \\ \frac{du(x)}{dx}\big|_{x=1} \end{bmatrix}$$

and compare with Problem 1 and 2.

4. If $U_4(x,s) = U_1(x,s) + ax + b$ where a and b are constants, we can have (10%)

$$u(s) = \frac{\partial U_4(x,s)}{\partial x}u(x)\bigg|_0^1 - U_4(x,s)\frac{du(x)}{dx}\bigg|_0^1 \tag{7}$$

Please derive the stiiffness matrix of K such that

$$K\mathbf{u} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}\begin{bmatrix} u(0) \\ u(1) \end{bmatrix} = \begin{bmatrix} \frac{du(x)}{dx}\big|_{x=0} \\ \frac{du(x)}{dx}\big|_{x=1} \end{bmatrix}$$

and compare with Problem 1, 2 and 3.