邊界元素法1999 第十二次作業

1. In the course, the auxilliary system is fundamental solution $U(x,s)$ which satisfies

$$\frac{d^2U(x,s)}{dx^2} = \delta(x-s) \tag{1}$$

where

$$U(x,s) = \begin{cases} \frac{1}{2}(x-s), & x > s, \\ -\frac{1}{2}(x-s), & x < s \end{cases} \tag{2}$$

Now we change to non-source auxilliary system, the boundary integral equation reduces to

$$u(s) = T(x,s)u(x)|_0^1 - U(x,s)\frac{du(x)}{dx}\bigg|_0^1 \;\rightarrow\; 0 = T(x)u(x)|_0^1 - U(x)\frac{du(x)}{dx}\bigg|_0^1 \tag{3}$$

where

$$U(x,s) \;\rightarrow\; U(x) = ax + b$$

By considering the two combinations for a and b, we can derive the stiffness matrix of $[K]$ such that

$$[K]\mathbf{u} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} u(0) \\ u(1) \end{bmatrix} = \begin{bmatrix} P_0 \\ P_1 \end{bmatrix}$$

where $P_0 = -t(0)$ and $P(1) = t(1)$, and compare the result in the course by using any a and b.

Generalized inverse:

$$[K][K]^{-1}[K] = [K]$$

$$[K][F][K] = [K]$$

$$[F][F]^{-1}[F] = [F]$$

$$[F][K][F] = [F]$$

$$[K] = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[F] = \begin{bmatrix} \frac{1}{4} & \frac{-1}{4} \\ \frac{-1}{4} & \frac{1}{4} \end{bmatrix}$$

Ref:

J. T. Chen and and S. R. Kuo, A nonsingular integral formulation for the Helmhotltz equations, Submitted, 1999.