邊界元素法1999 第十三次作業

1. In the course, we derive the fundamental solution in the exact form. The fundamental solution can be represented in degenerate form as follows:
Closed form:

$$U(s,x) = ln(r)$$

Degenerate form:

$$\begin{aligned} U(s,x) &= \ln r = \ln\sqrt{(\rho\cos(\phi) - R\cos(\theta))^2 + (\rho\sin(\phi) - R\sin(\theta))^2} \\ &= \begin{cases} U^i(s,x) = \ln R - \sum_{m=1}^{\infty} \frac{1}{m}(\frac{\rho}{R})^m \cos(m(\theta-\phi)),\ R > \rho \\ U^e(s,x) = \ln\rho - \sum_{m=1}^{\infty} \frac{1}{m}(\frac{R}{\rho})^m \cos(m(\theta-\phi)),\ \rho > R \end{cases} \end{aligned}$$

where $s = (R,\theta)$, $x = (\rho,\phi)$ and $r = \mid x - s \mid$.
Special case:

$$\begin{aligned} U(s,x) &= \ln r = \ln\sqrt{2 - 2\cos(\theta-\phi)} \\ &= \begin{cases} U^i(s,x) = -\sum_{m=1}^{\infty} \frac{1}{m}\cos(m(\theta-\phi)),\ R > \rho \\ U^e(s,x) = -\sum_{m=1}^{\infty} \frac{1}{m}\cos(m(\theta-\phi)),\ \rho > R \end{cases} \end{aligned}$$

where $\rho = R = 1$.
2. Plot the two contour figues for $U(s,x)$ in the shadow region in Fig.1. using exact form and degenerate form by summing N terms instead of infinite terms.
3. Discuss the effect of the number of terms, N and compare the contour plots using closed form and degenerate form.
4. Extend the degenerate form to T, L and M kernels.
Extend Laplace equation to Helholtz equation:
Closed form:

$$U(\mathbf{s},\mathbf{x}) = \frac{-i\pi H_0^{(1)}(kr)}{2}$$

Degenerate form:
Real part:

$$\begin{aligned} U(s,x) &= RealPart\{\frac{-i\pi H_0^{(1)}(kr)}{2}\} \\ &= \begin{cases} U^i(s,x) = \sum_{m=-\infty}^{\infty} \frac{\pi}{2} J_m(k\rho)\, Y_m(kR)\, \cos(m(\theta-\phi)),\ R > \rho \\ U^e(s,x) = \sum_{m=-\infty}^{\infty} \frac{\pi}{2} J_m(kR)\, Y_m(k\rho)\, \cos(m(\theta-\phi)),\ \rho > R \end{cases} \end{aligned}$$

Imaginary part:

$$\begin{aligned} U_I(s,x) &= ImaginaryPart\{\frac{-i\pi H_0^{(1)}(kr)}{2}\} \\ &= \begin{cases} U^i(s,x) = -\sum_{m=-\infty}^{\infty} \frac{\pi}{2} J_m(k\rho)\, J_m(kR)\, \cos(m(\theta-\phi)),\ R > \rho \\ U^e(s,x) = -\sum_{m=-\infty}^{\infty} \frac{\pi}{2} J_m(kR)\, J_m(k\rho)\, \cos(m(\theta-\phi)),\ \rho > R \end{cases} \end{aligned}$$