邊界元素法1999 第十四次作業

1. In the previous homework, we have plotted the fundamental solution, $U(s,x)$, in the closed and degenerate forms as follows:
Closed form:

$$U(s,x) = ln(r)$$

Degenerate form:

$$\begin{aligned} U(s,x) &= \ln r = \ln \sqrt{(\rho\cos(\phi) - R\cos(\theta))^2 + (\rho\sin(\phi) - R\sin(\theta))^2} \\ &= \begin{cases} U^i(s,x) = \ln R - \sum_{m=1}^{\infty} \frac{1}{m}(\frac{\rho}{R})^m \cos(m(\theta-\phi)),\ R > \rho \\ U^e(s,x) = \ln \rho - \sum_{m=1}^{\infty} \frac{1}{m}(\frac{R}{\rho})^m \cos(m(\theta-\phi)),\ \rho > R \end{cases} \end{aligned}$$

$$\begin{aligned} T(s,x) &= \frac{\partial U(s,x)}{\partial n(s)} \\ &= \begin{cases} T^i(s,x) = \frac{1}{R} + \sum_{m=1}^{\infty} \frac{\rho^m}{R^{m+1}} \cos(m(\phi-\theta)),\ R > \rho \\ T^e(s,x) = -\sum_{m=1}^{\infty} \frac{R^{m-1}}{\rho^m} \cos(m(\phi-\theta)),\quad \rho > R \end{cases} \end{aligned}$$

where $s = (R,\theta)$, $x = (\rho,\phi)$ and $r = \mid x - s \mid$.
2. Now, please plot the two contour figues for $T(s,x)$ with normal vector $\mathbf{n}(s) = \mathbf{e}_R$ at the fixed point $s = (R,\theta)$ in the shadow region in Fig.1 using exact form and degenerate form by summing N terms instead of infinite terms.
3. Discuss the effect of the number of terms, N.
4. Compare the results with the plot of $T((0,0),(x_1,x_2))$

$$T(s,x) = \frac{-y_i n_i}{r^2}$$

where

$$y_i = x_i - s_i$$

$$(s_1,s_2) = (0,0)$$

$$\mathbf{n}(s) = (0,1)$$

References

[1] 陳正宗與洪宏基，邊界元素法，第二版，新世界出版社，台北，頁 90 與 214，1992。