

邊界元素法1999 第十五次作業

1. In the text book of Chen and Hong, we have constructed the potentials induced by a line singularity from $(-1, 0)$ to $(1, 0)$. Please reconstruct the potentials induced by a arc singularity from 0° to 15° . using the degenerate kernels as follows:
2. In the text, we have determined the following potential functions.

$$P_U(x_1, x_2) = \int_{-1}^1 U(s_1, 0; x_1, x_2) ds_1$$

$$P_T(x_1, x_2) = \int_{-1}^1 T(s_1, 0; x_1, x_2) ds_1$$

$$P_L(x_1, x_2) = \int_{-1}^1 L(s_1, 0; x_1, x_2) ds_1$$

$$P_M(x_1, x_2) = \int_{-1}^1 M(s_1, 0; x_1, x_2) ds_1$$

3. Determine the following potential functions.

$$P_U(\rho, \phi) = \int_{0^\circ}^{15^\circ} U(R, \theta; \rho, \phi) d\theta$$

$$P_T(\rho, \phi) = \int_{0^\circ}^{15^\circ} T(R, \theta; \rho, \phi) d\theta$$

$$P_L(\rho, \phi) = \int_{0^\circ}^{15^\circ} L(R, \theta; \rho, \phi) d\theta$$

$$P_M(\rho, \phi) = \int_{0^\circ}^{15^\circ} M(R, \theta; \rho, \phi) d\theta$$

where the four degenerate kernels can be expressed:

$$U(s, x) = \begin{cases} U^i(s, x) = \ln R - \sum_{m=1}^{\infty} \frac{1}{m} \left(\frac{\rho}{R}\right)^m \cos(m(\phi - \theta)), & R > \rho \\ U^e(s, x) = \ln \rho - \sum_{m=1}^{\infty} \frac{1}{m} \left(\frac{R}{\rho}\right)^m \cos(m(\phi - \theta)), & \rho > R \end{cases} \quad (1)$$

$$T(s, x) = \begin{cases} T^i(s, x) = \frac{1}{R} + \sum_{m=1}^{\infty} \frac{\rho^m}{R^{m+1}} \cos(m(\phi - \theta)), & R > \rho \\ T^e(s, x) = -\sum_{m=1}^{\infty} \frac{R^{m-1}}{\rho^m} \cos(m(\phi - \theta)), & \rho > R \end{cases} \quad (2)$$

$$L(s, x) = \begin{cases} L^i(s, x) = -\sum_{m=1}^{\infty} \frac{\rho^{m-1}}{R^m} \cos(m(\phi - \theta)), & R > \rho \\ L^e(s, x) = \frac{1}{\rho} + \sum_{m=1}^{\infty} \frac{R^m}{\rho^{m+1}} \cos(m(\phi - \theta)), & \rho > R \end{cases} \quad (3)$$

$$M(s, x) = \begin{cases} M^i(s, x) = \sum_{m=1}^{\infty} \frac{m\rho^{m-1}}{R^{m+1}} \cos(m(\phi - \theta)), & R > \rho \\ M^e(s, x) = \sum_{m=1}^{\infty} \frac{mR^{m-1}}{\rho^{m+1}} \cos(m(\phi - \theta)), & \rho > R \end{cases} \quad (4)$$

References

- [1] 陳正宗與洪宏基，邊界元素法，第二版，新世界出版社，台北，頁 90 與 214，