# 邊界元素法1999 第二次作業

1. In the course, we derive the fundamental solution in the exact form. The fundamental solution can be represented in degenerate form as follows:
**Closed form:**
$$U(s,x) = ln(r)$$
**Degenerate form:**
$$\begin{aligned} U(s,x) = \ln r = \ln\sqrt{(\rho\cos(\phi) - R\cos(\theta))^2 + (\rho\sin(\phi) - R\sin(\theta))^2} \\ = \begin{cases} U^i(s,x) = \ln R - \sum_{m=1}^{\infty} \frac{1}{m}(\frac{\rho}{R})^m \cos(m(\theta-\phi)),\ R > \rho \\ U^e(s,x) = \ln\rho - \sum_{m=1}^{\infty} \frac{1}{m}(\frac{R}{\rho})^m \cos(m(\theta-\phi)),\ \rho > R \end{cases} \end{aligned}$$
where $s = (R,\theta)$, $x = (\rho,\phi)$ and $r = \mid x - s \mid$.
Special case:
$$\begin{aligned} U(s,x) = \ln r = \ln\sqrt{2 - 2\cos(\theta-\phi)} \\ = \begin{cases} U^i(s,x) = -\sum_{m=1}^{\infty} \frac{1}{m}\cos(m(\theta-\phi)),\ R > \rho \\ U^e(s,x) = -\sum_{m=1}^{\infty} \frac{1}{m}\cos(m(\theta-\phi)),\ \rho > R \end{cases} \end{aligned}$$
where $\rho = R = 1$.
2. Plot the two contour figues for $U(s,x)$ in the shadow region in Fig.1 using exact form and degenerate form by summing $N$ terms instead of infinite terms.
3. Discuss the effect of the number of terms, $N$, and compare the contour plots using closed form and degenerate form.
4. Extend the degenerate form to $T$, $L$ and $M$ kernels.