邊界元素法1999 第九次作業

1. In the book of Chen and Zhou, the Calderon projectors are defined by

$$[C_1] = \begin{bmatrix} \frac{1}{2}[I]-[T] & [U] \\ -[M] & \frac{1}{2}[I]+[L] \end{bmatrix} \tag{1}$$

$$[C_2] = \begin{bmatrix} \frac{1}{2}[I]+[T] & -[U] \\ +[M] & \frac{1}{2}[I]-[L] \end{bmatrix} \tag{2}$$

The Calderon identities are

$$[C_1]+[C_2]=[I]$$

$$[C_1]^2=[C_1]$$

$$[C_1][C_2]=[C_2][C_1]=0$$

The relations between potential opeartors are shown below:

$$-\frac{1}{4}[I]+[T]^2-[U][M]=0$$

$$[U][L]=[T][U]$$

$$[M][T]=[L][M]$$

$$-\frac{1}{4}[I]+[L]^2-[M][U]=0$$

(1). Derive the above identities by using

$$[U^i]\{t\}=[T^i]\{u\}$$

$$[L^i]\{t\}=[M^i]\{u\}$$

where

$$[U^i]=[U^e]=[U]$$

$$[T]=[T^i]+\frac{1}{2}[I]=[T^e]-\frac{1}{2}[I]$$

$$[L]=[L^i]-\frac{1}{2}[I]=[L^e]+\frac{1}{2}[I]$$

$$[M^i]=[M^e]=[M]$$

(2). Using the circulants for a two-dimesional circular domain, please prove the Calderon projector property.