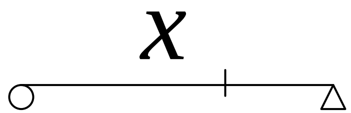
格林函數(Green’s function)

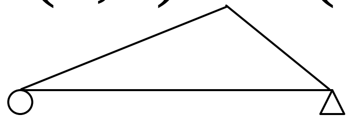
- 格林函數 $G(x,s)$

  在 s 處施加一外力項，在 x 處所得到之反應 $G(x,s)$

- 影響線(influence line)

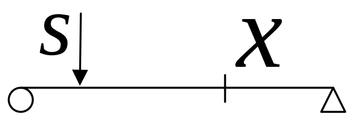


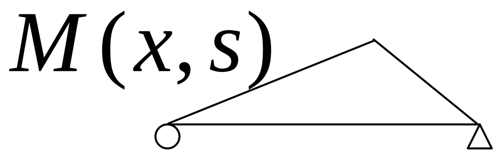
$$M(x,s) = G(x,s)$$



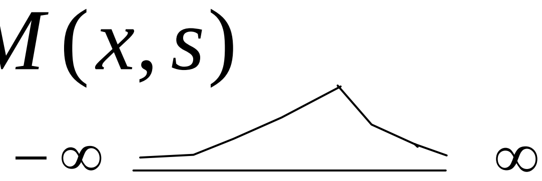
基本解(Fundamental solution)

- 格林函數





- 基本解(自由空間格林函數)



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二維拉普拉斯方程

- 欲解系統

$$G.E.\quad \nabla^2 u = 0 \qquad u$$

- 輔助系統

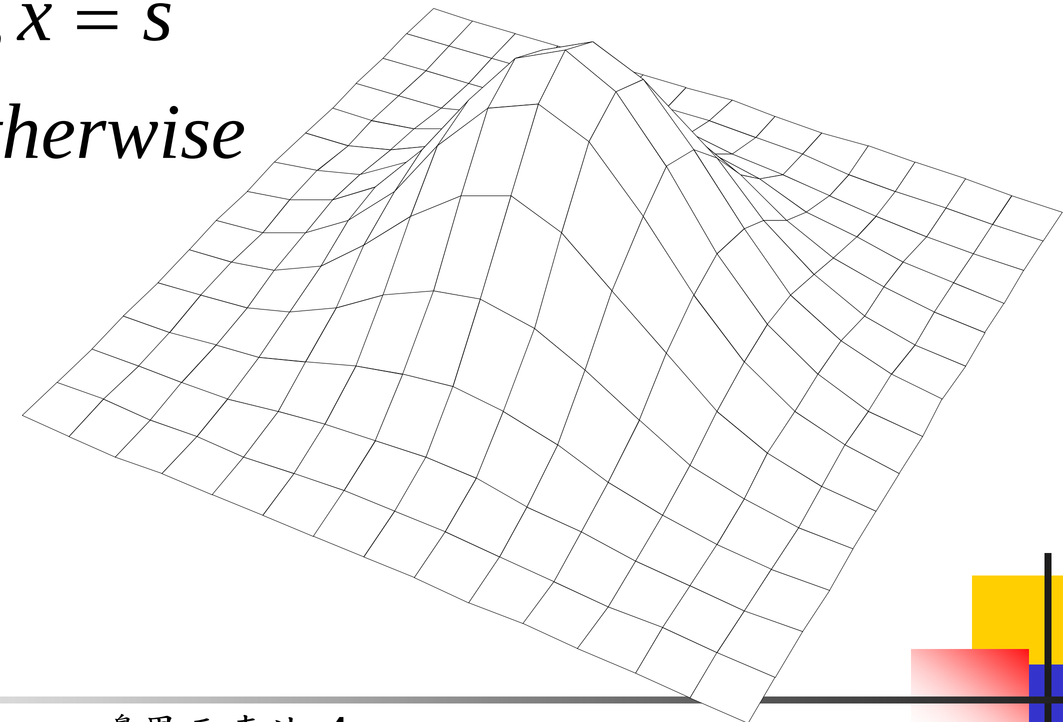
$$G.E.\quad \nabla^2 U(x,s) = 2\pi\delta(x-s) \qquad U(x,s)$$

Delta Function $\delta(x)$

- 1單位之集中力

$$\delta(x-s)=\begin{cases}\infty & ,x=s\\ 0 & ,otherwise\end{cases}$$

$$\int_D 2\pi\delta(x-s)\,dD=1$$



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散度定理

$$\iint \nabla \cdot \Phi \; dD = \int n \cdot \Phi \; dB$$

$$\Phi = u\nabla v$$

$$\iint u\nabla^2 v \;\; dD = \int_B u \frac{\partial v}{\partial n} dB - \iint \nabla u \nabla v \; dD$$

$$\Phi = v\nabla u$$

$$\iint v\nabla^2 u \;\; dD = \int_B v \frac{\partial u}{\partial n} dB - \iint \nabla u \nabla v \; dD$$

功能互换

- 格林第三定理

$$\iint (u\nabla^2 v - v\nabla^2 u)\, dD = \int_B (u\frac{\partial v}{\partial n} - v\frac{\partial u}{\partial n})\, dB$$

將 $u = u,\ \ v = U(x,s),$

$\nabla^2 u = 0, \quad \nabla^2 U(x,s) = 2\pi\delta(x-s)$ 代入

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積分方程式

$$2\pi u(s) = \int_B \frac{\partial U(x,s)}{\partial n_x} u(x) dB(x) - \int_B U(x,s) \frac{\partial u(x)}{\partial n_x} dB(x)$$

$$s \in D, \quad x \in B,$$

- 域內點積分方程式(DIE)

$$2\pi u(x) = \int_B T(s,x) u(s) dB(s) - \int_B U(s,x) t(s) dB(s)$$

$$x \in D, \quad s \in B, \quad T(s,x) = \frac{\partial U(s,x)}{\partial n_s}, \quad t(s) = \frac{\partial u(s)}{\partial n_s}$$

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邊界積分方程式(BIE)

- 奇異積分方程

$$\pi u(x) = C.P.V.\int_B T(s,x)u(s)dB(s) - R.P.V.\int_B U(s,x)t(s)dB(s), \quad x \in B$$

- 超奇異積分方程

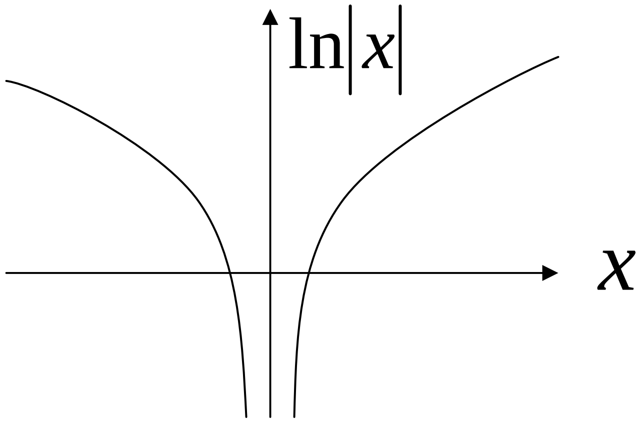
$$\pi t(x) = H.P.V.\int_B M(s,x)u(s)dB(s) - C.P.V.\int_B L(s,x)t(s)dB(s), \quad x \in B$$

$$U(s,x) = \ln(r), \qquad T(s,x) = \frac{\partial U(s,x)}{\partial n_s},$$

$$L(s,x) = \frac{\partial U(s,x)}{\partial n_x}, \quad M(s,x) = \frac{\partial^2 U(s,x)}{\partial n_s \partial n_x}$$

弱奇異積分

- 黎曼主值 $R.P.V.$



$$\int_{-1}^{1} \ln|x|\, dx = (x\ln|x| - x)\Big|_{-1}^{1} = -2$$

奇異，超奇異積分

- 柯西主值 $C.P.V.$

$$\int_{-1}^{1}\frac{1}{x}dx=\lim_{\varepsilon\to 0}\int_{-1}^{-\varepsilon} \quad +\int_{\varepsilon}^{1}\frac{1}{x}dx=0$$

- 阿達馬主值 $H.P.V.$

$$\int_{-1}^{1}\frac{1}{x^2}dx=\lim_{\varepsilon\to 0}\int_{-1}^{-\varepsilon} \quad +\int_{\varepsilon}^{1}\frac{1}{x^2}dx-\frac{2}{\varepsilon}=-2$$

離散化

- 直接法

$$\pi u(x) = C.P.V.\int_B T(s,x)u(s)dB(s) - R.P.V.\int_B U(s,x)t(s)dB(s), \quad x \in B$$

$$[U]\{t\} = [T]\{u\}$$

$$\pi u(x) = C.P.V.\int_B T(s,x)u(s)dB(s) - R.P.V.\int_B U(s,x)t(s)dB(s), \quad x \in B$$

$$[L]\{t\} = [M]\{u\}$$

BEM解題流程

控制方程(G.E.) $\pounds(u)=p$

邊界條件(B.C.) $B(u)=q$

→

求基本解

$\pounds(U)=\delta(r)$

↓

邊界積分方程(BIE)

←

離散化線性代數方程

↓

求邊界值

解域內物理量 →

域內積分方程(DIE)

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