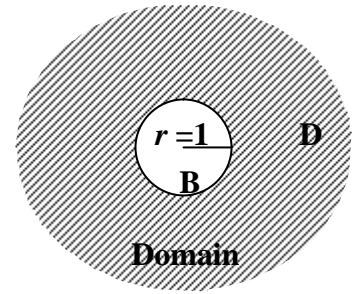Trefftz method

$$\nabla^2 u(x) = 0, \quad x \in D$$

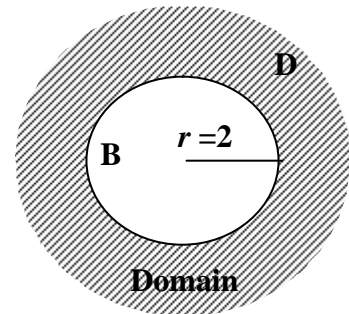$$u(x) = \cos(3q), \quad x \in B$$

(a) By choosing T-complete set, $\ln r$, $\dfrac{1}{r}\cos q$, $\dfrac{1}{r}\sin q$, $\dfrac{1}{r^2}\cos 2q$, $\dfrac{1}{r^2}\sin 2q$, find

the Trefftz solution using the point collocation approach.

(b) By choosing T-complete set, $\ln r$, $\dfrac{1}{r}\cos q$, $\dfrac{1}{r}\sin q$, $\dfrac{1}{r^2}\cos 2q$, $\dfrac{1}{r^2}\sin 2q$,

$\dfrac{1}{r^3}\cos 3q$, $\dfrac{1}{r^3}\sin 3q$, $\dfrac{1}{r^4}\cos 4q$, $\dfrac{1}{r^4}\sin 4q$, find the Trefftz solution using the

point collocation approach.

Extend the HW2 to exterior problem (Two cases $r=1$ and $r=2$)

$$u(x) = \ln 2 + \frac{1}{8}\cos(3q), \quad x \in B$$

Ans   Exact solution $u(r, q) = \ln r + \dfrac{1}{r^3}\cos 3q$

References:
 1   J.T. Chen, I.L. Chen, C.S. Wu, On the equivalence of MFS and Trefftz method
     for Laplace problems, Global Chinese Workshop on Boundary Element and
     Meshless Methods, Qinhuandao, 2003.