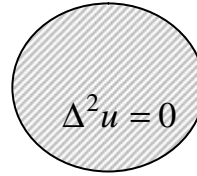


程式 82 Degenerate scale for biharmonic operator

Example 1:

G. E.: $\Delta^2 u = 0$

B. C.:
$$\begin{cases} u = x^4 - y^4 \\ \frac{\partial u}{\partial n} = 4(x^3 n_x - y^3 n_y) \end{cases}$$

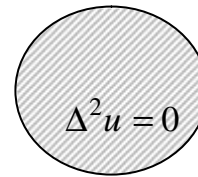


Exact solution: $u(x, y) = x^4 - y^4$

Example 2:

G. E.: $\Delta^2 u = 0$

B. C.:
$$\begin{cases} u = \frac{-1}{4} \\ \frac{\partial u}{\partial n} = \frac{-1}{2}(1 + \cos q) \end{cases}$$

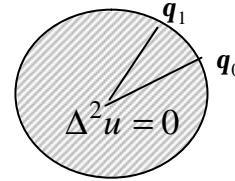


Exact solution: $u(r, q) = \frac{1}{4}(1 - r^2)(1 + r \cos q) - \frac{1}{4}$

Example 3:

G. E.: $\Delta^2 u = 0$

B. C.:
$$\frac{\partial u}{\partial n} = \begin{cases} -1, & q_0 < q < q_1 \\ 0, & q_1 < q < 2p + q_0 \end{cases}$$



Exact solution: $u(r, q) = \frac{1}{2p}(1 - r^2)[g + \arctan(\frac{1+r}{1-r} \tan \frac{q_1 - q}{2}) - \arctan(\frac{1+r}{1-r} \tan \frac{q_0 - q}{2})]$

where $g = \begin{cases} 0, & q_1 - p < q < q_0 + p \\ p, & q_0 + p < q < q_1 + p \end{cases}$

By using the null-field integral equations in conjunction with the degenerate kernels and Fourier series, derive the analytical solution.

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2. Mills R. D., 1977, Computing internal viscous flow problems for the circle by integral methods, J. Fluid Mech., Vol.12, pp.609-624.