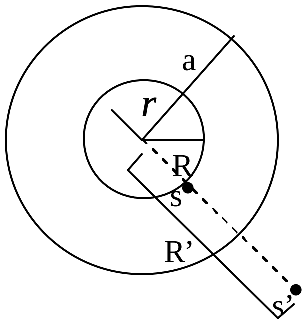
Derivation of the Green's function using the series form (degenerate kernel)

$$\ln r = \begin{cases} \ln r - \sum_{m=1}^{\infty} \frac{1}{m} (\frac{R}{r})^m \cos[m(q - f)], \ when \ \ r > R \\ \ln R - \sum_{m=1}^{\infty} \frac{1}{m} (\frac{r}{R})^m \cos[m(q - f)], \ when \ \ r < R \end{cases}$$

$$R < a < R^{'}$$

$$\text{其中}, R^{'} = \frac{a^2}{R}$$



當 $0 < r < R$ 時,

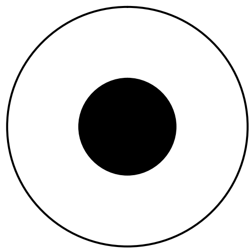
$$U_p(x,s) = \ln|x-s| - \ln|x-s^{'}| + \ln a - \ln R$$

$$= \{\ln R - \sum_{m=1}^{\infty} \frac{1}{m} (\frac{r}{R})^m \cos[m(q-f)]\}$$

$$- \{\ln(\frac{a^2}{R}) - \sum_{m=1}^{\infty} \frac{1}{m} (\frac{r\ R}{a^2})^m \cos[m(q-f)]\}$$

$$+ \ln a - \ln R$$

$$= \ln(\frac{R}{a}) - \sum_{m=1}^{\infty} \frac{1}{m} [(\frac{r}{R})^m - (\frac{r\ R}{a^2})^m] \cos[m(q-f)]$$



當 $R < r < a$ 時,

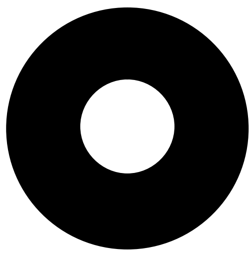
$$U_p(x,s) = \ln|x-s| - \ln|x-s^{'}| + \ln a - \ln R$$

$$= \{\ln r - \sum_{m=1}^{\infty} \frac{1}{m} (\frac{R}{r})^m \cos[m(q-f)]\}$$

$$- \{\ln(\frac{a^2}{R}) - \sum_{m=1}^{\infty} \frac{1}{m} (\frac{r\ R}{a^2})^m \cos[m(q-f]\}$$

$$+ \ln a - \ln R$$

$$= \ln(\frac{r}{a}) - \sum_{m=1}^{\infty} \frac{1}{m} [(\frac{R}{r})^m - (\frac{r\ R}{a^2})]^m \cos[m(q-f)]$$



$$U_p(x,s) = \begin{cases} \ln(\frac{R}{a}) - \sum_{m=1}^{\infty} \frac{1}{m} [(\frac{r}{R})^m - (\frac{r\ R}{a^2})^m] \cos[m(q-f)], \ 0 < r < R \\ \ln(\frac{r}{a}) - \sum_{m=1}^{\infty} \frac{1}{m} [(\frac{R}{r})^m - (\frac{r\ R}{a^2})^m] \cos[m(q-f)], \ R < r < a \end{cases}$$

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