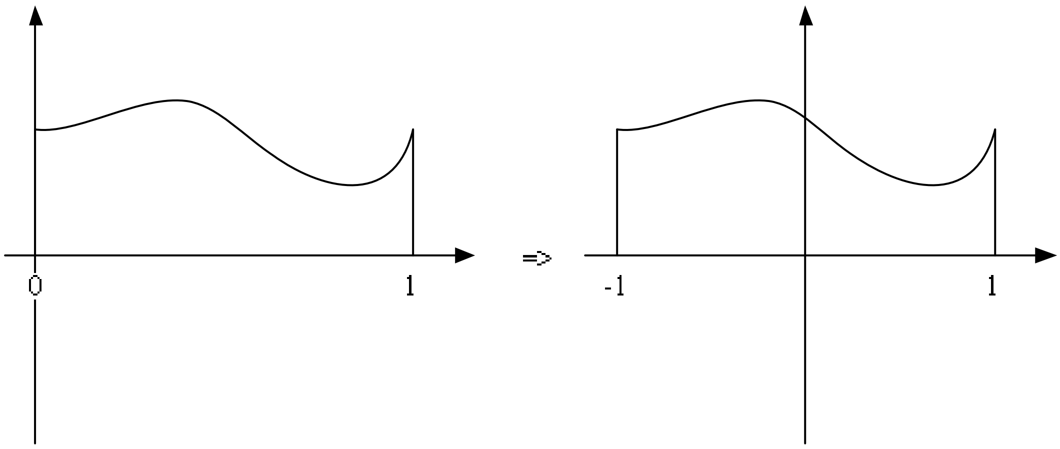
$$U=\int_0^1 \mathrm{Log}\left[\sqrt{(1-s)^2+(0.5)^2}\right] ds = -0.334853865444584994$$

$$T=\int_0^1 \frac{0.5}{(1-s)^2+0.5^2}\, ds = 1.1071487177940904$$

$$L=\int_0^1 \frac{1-s}{(1-s)^2+0.5^2}\, ds = 0.8047189562170501$$

$$M=\int_0^1 -\frac{2*0.5*(1-s)}{((1-s)^2+0.5^2)^2}\, ds = -1.6$$

高斯積分



$$v = as + b$$

$$\begin{cases} 1 = a + b \\ -1 = b \end{cases}$$

$$a = 2$$

$$v = 2s - 1$$

$$s = \frac{v+1}{2}$$

$$ds = \frac{1}{2} dv$$

$$U = \int_0^1 \ln(\sqrt{(1-s)^2 + 0.5^2})ds$$
$$= \sum_{i=1}^{n} W_i \frac{1}{2} \ln(\sqrt{(1-\frac{v_i+1}{2})^2 + 0.5^2})$$

$$T=\int_0^1 \frac{0.5}{(1-s)^2+0.5^2}\,ds$$

$$=\sum_{i=1}^{n} W_i \frac{1}{2}\frac{0.5}{(1-\frac{v_i+1}{2})^2+0.5^2}$$

$$L=\int_0^1 \frac{1-s}{(1-s)^2+0.5^2}\,ds$$

$$=\sum_{i=1}^{n} W_i \frac{1}{2}\frac{1-\frac{v_i+1}{2}}{(1-\frac{v_i+1}{2})^2+0.5^2}$$

$$L=\int_0^1 \frac{-2\times 0.5\times(1-s)}{\left((1-s)^2+0.5^2\right)^2}\,ds$$

$$=\sum_{i=1}^{n} W_i \frac{1}{2}\frac{-2\times 0.5\times\left(1-\frac{v_i+1}{2}\right)}{\left((1-\frac{v_i+1}{2})^2+0.5^2\right)^2}$$

程式結果

輸入高斯積分點點數

3

	1	2	3
x=	0.774597	0.000000	-0.774597
w=	0.555556	0.888889	0.555556

U	T	L	M
-0.3346045	1.107034	0.8012233	-1.571585

輸入高斯積分點點數

5

	1	2	3	4	5
x=	0.906180	0.538469	0.000000	-0.538469	-0.906180
w=	0.236927	0.478629	0.568889	0.478629	0.236927

U	T	L	M
-0.3348573	1.107174	0.8047636	-1.600755

	1
x=	0
w=	2

	1	2
x=	0.57735	-0.57735
w=	1	1

	1	2	3
x=	0.774597	0	-0.7746
w=	0.555556	0.888889	0.555556

	1	2	3	4
x=	0.861136	0.339981	-0.33998	-0.86114
w=	0.347855	0.652145	0.652145	0.347855

	1	2	3	4	5
x=	0.90618	0.538469	0	-0.53847	-0.90618
w=	0.236927	0.478629	0.568889	0.478629	0.236927

	1	2	3	4	5	6
x=	0.93247	0.661209	0.238619	-0.23862	-0.66121	-0.93247
w=	0.171324	0.360761	0.467914	0.467914	0.360761	0.171324

	1	2	3	4	5	6	7
x=	0.949108	0.741531	0.405845	0	-0.40585	-0.74153	-0.94911
w=	0.129485	0.279705	0.38183	0.417959	0.38183	0.279705	0.129485

	1	2	3	4	5	6	7	8
x=	0.96029	0.796667	0.525532	0.183435	-0.18344	-0.52553	-0.79667	-0.96029
w=	0.101229	0.222381	0.313707	0.362684	0.362684	0.313707	0.222381	0.101229

	1	2	3	4	5	6	7	8	9
x=	0.96816	0.836031	0.613371	0.324253	0	-0.32425	-0.61337	-0.83603	-0.96816
w=	0.081274	0.180648	0.260611	0.312347	0.330239	0.312347	0.260611	0.180648	0.081274

	1	2	3	4	5	6	7	8	9	10
x=	0.973907	0.865063	0.67941	0.433395	0.148874	-0.14887	-0.4334	-0.67941	-0.86506	-0.97391
w=	0.066671	0.149451	0.219086	0.269267	0.295524	0.295524	0.269267	0.219086	0.149451	0.066671