

Derivation of Poisson kernel

$$x = (\mathbf{r}, \mathbf{f}), \quad s = (R, \mathbf{q})$$

$$2\mathbf{p}u(s) = \int_B T_p(x, s)u(x)dB(x) - \int_B U_p(x, s)t(x)dB(x) \quad \leftarrow \text{域內內點的邊界基分方程}$$

若 $\mathbf{r} = a$ 時, $U_p(x, s) = 0$

$$2\mathbf{p}u(s) = \int_B T_p(x, s)u(x)dB(x)$$

$$= \int_0^{2\mathbf{p}} T_p(\mathbf{r}, \mathbf{f}; R, \mathbf{q}; \frac{a^2}{R}, \mathbf{q})f(\mathbf{f})a d\mathbf{f}$$

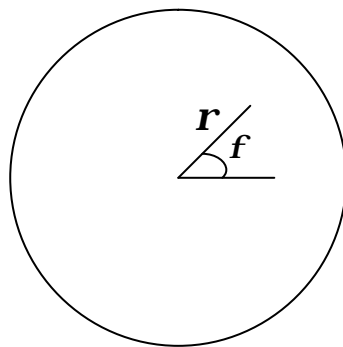
$\mathbf{r} = a$ 代入

$$u(R, \mathbf{q}) = \frac{1}{2\mathbf{p}} \int_0^{2\mathbf{p}} T_p(R, \mathbf{f}; R, \mathbf{q}; \frac{a^2}{R}, \mathbf{q})f(\mathbf{f})a d\mathbf{f}$$

$$= \frac{1}{2\mathbf{p}} \int_0^{2\mathbf{p}} \frac{a^2 - R^2}{a[a^2 + R^2 - 2aR\cos(\mathbf{f} - \mathbf{q})]} f(\mathbf{f})a d\mathbf{f}$$

$$= \frac{1}{2\mathbf{p}} \int_0^{2\mathbf{p}} \frac{a^2 - R^2}{a^2 + R^2 - 2aR\cos(\mathbf{f} - \mathbf{q})} f(\mathbf{f})d\mathbf{f}$$

$$u(\mathbf{r}, \mathbf{f}) = \frac{1}{2\mathbf{p}} \int_0^{2\mathbf{p}} \frac{a^2 - r^2}{a^2 + r^2 - 2ar\cos(\mathbf{f} - \mathbf{q})} f(\mathbf{q})d\mathbf{q}$$



$$f(\mathbf{q}) = u(1, \mathbf{q})$$