Derivation of Poisson integral formula

(classical method)

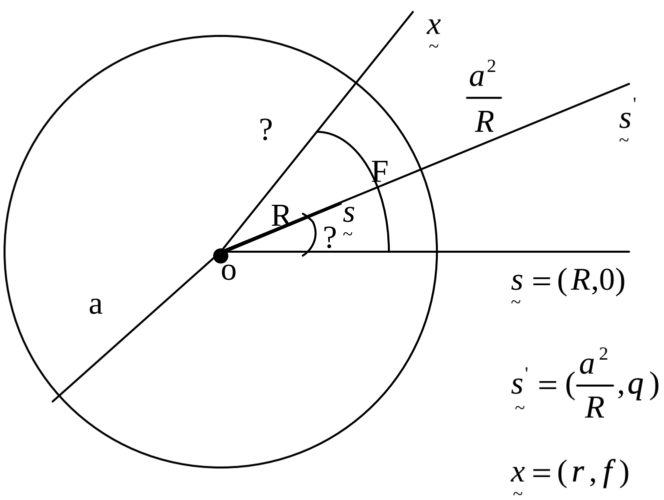
$$2p\,u(s) = \int_B T(x,s)u(x)dB(x) - \int_B U(x,s)t(x)dB(x)$$

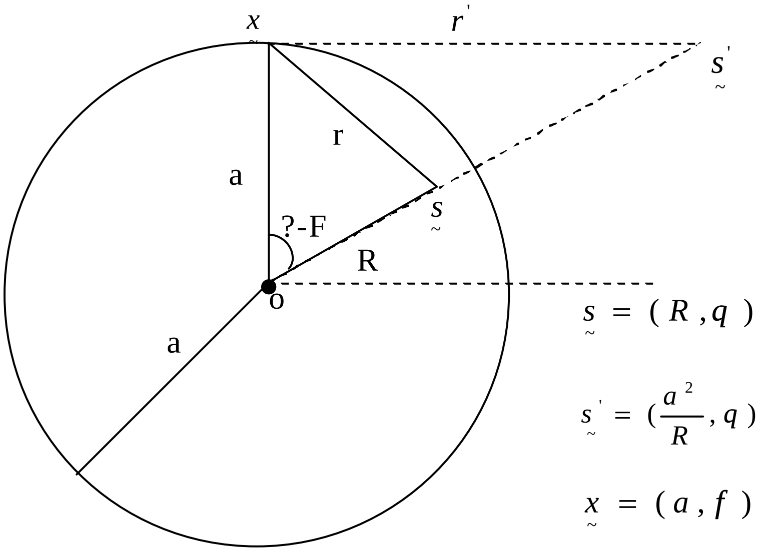
$$2p\;u(s) = \int_B T(x;s,s^{'})u(x)dB(x) - \int_B U(x;s,s^{'})t(x)dB(x)$$

where $U(x,s)$ **and** $U(x;s,s^{'})$ **are the fundamental solution and the Green's function**

若我們可選取輔助系統 $U(x;s,s^{'})=0$ **, x on B 上式可簡化為**

$$2pu(s) = \int_B T(x;s,s^{'})u(x)dB(x)$$





$$U(x;s,s^{'}) = \ln|x-s^{'}| - \ln|x-s| - \ln(\frac{R}{a})$$

$$\ln|x-s| = \ln r = \ln\sqrt{a^2+R^2-2aR\cos(f-q)}$$

$$\ln|x-s^{'}| = \ln r^{'} = \ln\sqrt{a^2+(\frac{a^2}{R})^2-2a(\frac{a^2}{R})\cos(f-q)}$$

$$\ln r^{'} - \ln r = \ln\frac{\frac{a}{R}\sqrt{R^2+a^2-2aR\cos(f-q)}}{\sqrt{a^2+R^2-2aR\cos(f-q)}} = \ln(\frac{a}{R})$$

$$U(x;s,s^{'}) = \ln|x-s^{'}| - \ln|x-s| - \ln(\frac{a}{R})$$

$$\nabla_x^2 U(x;s,s^{'}) = d(x-s) - d(x-s^{'})$$

$$U^i = \ln r - \sum_{m=1}^{\infty}\frac{1}{m}(\frac{R}{r})^m \cos m(q-f)$$

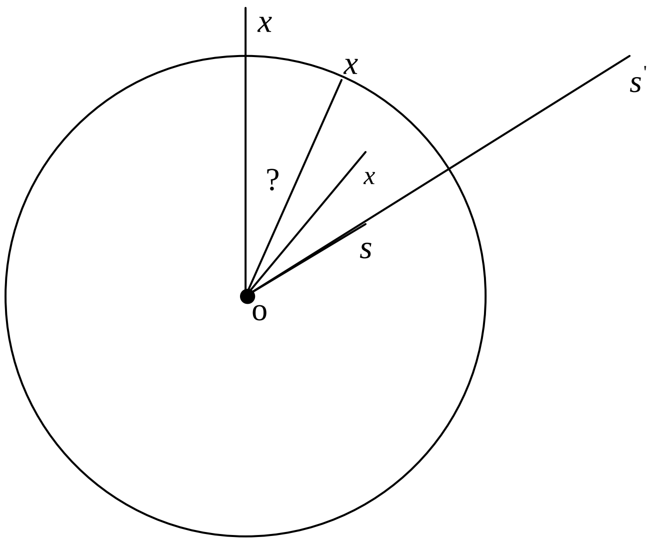
$$U^e = \ln R^{'} - \sum_{m=1}^{\infty}\frac{1}{m}(\frac{r}{R^{'}})^m \cos m(q-f)$$

$x \rightarrow Boundary$ $\qquad$ $x = (a, f)$

$$U^i = \ln a - \sum_{m=1}^{\infty}\frac{1}{m}(\frac{R}{a})^m \cos m(q-f)$$

$$U^e = \ln R^{'} - \sum_{m=1}^{\infty}\frac{1}{m}(\frac{a}{R^{'}})^m \cos m(q-f)$$

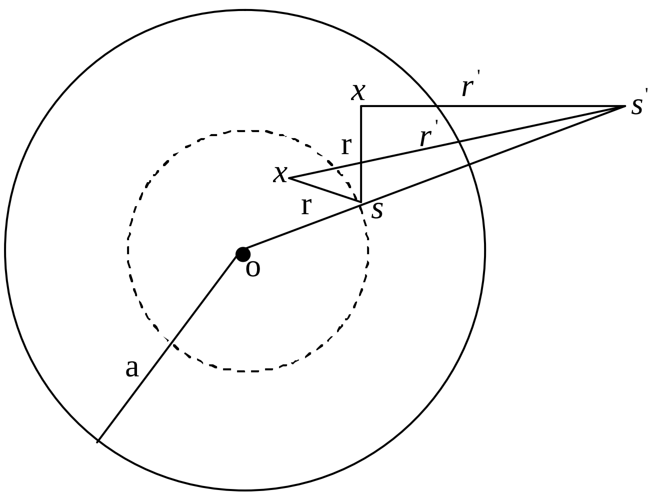
退化核想法



若吾人選取 $\frac{R}{a}=\frac{a}{R'}$ **時，上式兩式相減**

$$U^i - U^e = \ln R' - \ln a = \ln(\frac{a^2}{R}) - \ln a = \ln(\frac{a}{R})$$

Express the Green's function in terms of degenerate kernel



Nov.26/2003 **佳聰製表**(poisson.doc)