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1. $$\frac{d^4 U(x,s)}{d x^4} = \delta(x-s), \quad -\infty < x < \infty$$

(1) Is $U(x,s)$ singular

(2) Is $U(x,s)$ symmetric

(3) Is $U(x,s)$ degenerate form

(4) 3D Plot $U(x,s)$ and contour Plot

ANS

Fourier Transform

$$\int_{-\infty}^{\infty} \frac{d^4 U(x,s)}{d x^4} e^{ikx} dx = \int_{-\infty}^{\infty} \delta(x-s) e^{ikx} dx$$

$$\rightarrow k^4 u(k,s) = e^{-iks}$$

$$\rightarrow u(k,s) = \frac{e^{-iks}}{k^4}$$

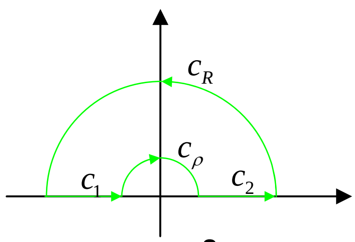
$\rightarrow$ Inverse Fourier Transform

$$\rightarrow U(x,s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} u(k,s) e^{ikx} dk = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{k^4} e^{ik(x-s)} dk$$

$$\text{Let } f(z,s) = \frac{e^{iz(x-s)}}{(z^4)}$$

$$\int_{-\infty}^{\infty} f(z,s)\, dz = \int_{C_1} + \int_{C_2} + \int_{C_\rho} + \int_{C_R} f(z,s)\, dz = 0$$

$$\rightarrow \int_{C_1} + \int_{C_2} + \int_{C_\rho} f(z,s)\, dz = 0$$



(a) $x > s$

$e^{iz(x-s)}$ 在0的Taylor級數

$$e^{iz(x-s)} = 1 + i(x-s)z - \frac{(x-s)^2}{2} z^2 - i\frac{(x-s)^3}{6} z^3 + \ldots$$

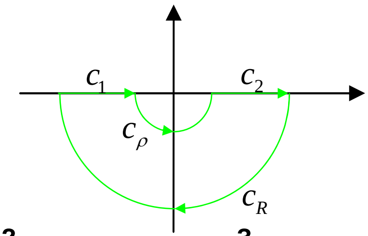
$$\int_{C_\rho} \frac{e^{iz(x-s)}}{(z^4)} dz = \int_{\pi}^{0} \left[\frac{1}{z^4} + i\frac{(x-s)}{z^3} - \frac{(x-s)^2}{2z^2} - i\frac{(x-s)^3}{6z} + \ldots\right] dz$$

$$\rightarrow \int_{C_\rho} \frac{e^{iz(x-s)}}{(z^4)}\,dz = \frac{-2}{3\rho^3} + \frac{(x-s)^2}{\rho} - \frac{\pi}{6}(x-s)^3$$

$$\int_{C_1} + \int_{C_2} + \int_{C_\rho} f(z,s)\,dz = 0$$

$$\rightarrow \frac{1}{2\pi}\left[\int_{C_1} + \int_{C_2} f(z,s)\,dz - \frac{2}{3\rho^3} + \frac{(s-x)^2}{\rho} - \frac{\pi}{6}(x-s)^3\right] = 0$$

$$\rightarrow \frac{1}{2\pi}\left[\int_{C_1} + \int_{C_2} f(z,s)\,dz - \frac{2}{3\rho^3} + \frac{(s-x)^2}{\rho}\right] = \frac{1}{12}(x-s)^3$$



(b) $x < s$

$e^{-iz(s-x)}$ 在0的Taylor級數

$$e^{-iz(s-x)} = 1 - i(x-s)z - \frac{(x-s)^2}{2}z^2 + i\frac{(x-s)^3}{6}z^3 + \ldots$$

$$\int_{C_\rho} \frac{e^{-iz(s-x)}}{(z^4)}\,dz = \int_\pi^0 \left[\frac{1}{z^4} - i\frac{(x-s)}{z^3} - \frac{(x-s)^2}{2z^2} + i\frac{(x-s)^3}{6z} + \ldots\right] dz$$

$$\rightarrow \int_{C_\rho} \frac{e^{-iz(s-x)}}{(z^4)}\,dz = \frac{-2}{3\rho^3} + \frac{(x-s)^2}{\rho} + \frac{\pi}{6}(x-s)^3$$

$$\int_{C_1} + \int_{C_2} + \int_{C_\rho} f(z,s)\,dz = 0$$

$$\rightarrow \frac{1}{2\pi}\left[\int_{C_1} + \int_{C_2} f(z,s)\,dz - \frac{2}{3\rho^3} + \frac{(x-s)^2}{\rho} + \frac{\pi}{6}(x-s)^3\right] = 0$$

$$\rightarrow \frac{1}{2\pi}\left[\int_{C_1} + \int_{C_2} f(z,s)\,dz - \frac{2}{3\rho^3} + \frac{(x-s)^2}{\rho}\right] = \frac{1}{12}(s-x)^3$$

$$\rightarrow \frac{1}{12}(s-x)^3 \quad x < s$$

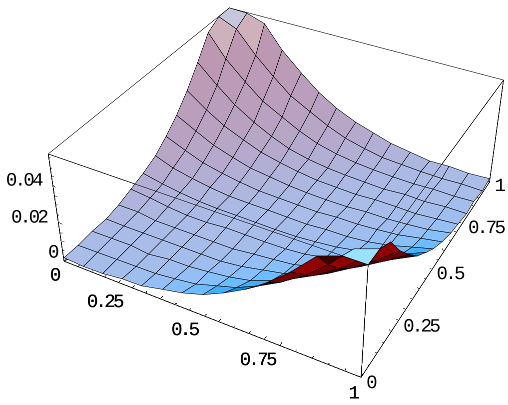
$$U(x,s)\begin{cases} \frac{1}{12}(x-s)^3 & x > s \\ \frac{1}{12}(s-x)^3 & x < s \end{cases}$$

(1) regular

(2) symmetric

(3) degenerate form

Plot3D



Contour Plot