(1),

$$U(s,x) = \ln[\rho] - \sum_{m=1}^{\infty} \frac{1}{m} \left(\frac{R}{\rho}\right)^m \cos[m(\theta-\phi)] \quad , \quad R > \rho$$

$$\frac{\partial U}{\partial \rho} = L(s,x) = \frac{1}{\rho} - \sum_{m=1}^{\infty} \frac{-m}{m} \frac{R^m}{\rho^{m+1}} \cos[m(\theta-\phi)] \quad , \quad R > \rho$$

$$\frac{\partial^2 U}{\partial R\, \partial \rho} = M(s,x) = \sum_{m=1}^{\infty} m \frac{R^{m-1}}{\rho^{m+1}} \cos[m(\theta-\phi)] \quad , \quad R > \rho$$

$$0 = \int_B M(s,x)\, u(s)\, dB(s) - \int_B L(s,x)\, t(s)\, dB(s)$$

$$= \int_0^{2\pi} \left[\sum_{m=1}^{\infty} m \frac{R^{m-1}}{\rho^{m+1}} \cos[m(\theta-\phi)]\right] [\cos(2\theta)]\, d\theta -$$

$$\int_0^{2\pi} \left[\frac{1}{\rho} + \sum_{m=1}^{\infty} \frac{R^m}{\rho^{m+1}} \cos[m(\theta-\phi)]\right] \left[P_0 + \sum_{n=1}^{\infty} (P_n \cos[n\theta] + q_n \sin[n\theta])\right] d\theta$$

$$= \int_0^{2\pi} \left[\sum_{m=1}^{\infty} m \frac{R^{m-1}}{\rho^{m+1}} (\cos[m\theta]\cos[m\phi] + \sin[m\theta]\sin[m\phi])\right] [\cos(2\theta)]\, d\theta$$

$$- \int_0^{2\pi} \left[\frac{1}{\rho} + \sum_{m=1}^{\infty} \frac{R^m}{\rho^{m+1}} (\cos[m\theta]\cos[m\phi] + \sin[m\theta]\sin[m\phi])\right] \Big[$$

$$P_0 + \sum_{n=1}^{\infty} (P_n \cos[n\theta] + q_n \sin[n\theta])\Big]\, d\theta$$

$(R = 1)$

$$= \int_0^{2\pi} \frac{2}{\rho^2} \cos^2[2\theta] \cos[2\phi]\, d\theta -$$

$$\int_0^{2\pi} \left[\frac{P_0}{\rho} + \sum_{n=1}^{\infty} \left(\frac{P_n}{\rho^{m+1}} \cos^2[n\theta]\cos[n\phi] + \frac{q_n}{\rho^{m+1}} \sin^2[n\theta]\sin[n\phi]\right)\right] d\theta$$

$$= \frac{2\pi}{\rho^2} \cos[2\phi] - \frac{2\pi P_0}{\rho} - \sum_{n=1}^{\infty} \frac{P_n}{\rho^{m+1}} \cos[n\phi] \int_0^{2\pi} \cos^2[n\theta]\, d\theta -$$

$$\sum_{n=1}^{\infty} \frac{q_n}{\rho^{m+1}} \sin[n\phi] \int_0^{2\pi} \sin^2[n\theta]\, d\theta$$

$$= \frac{2\pi}{\rho^2} \cos[2\phi] - \pi \sum_{n=1}^{\infty} \frac{P_n}{\rho^{m+1}} \cos[n\phi] - \pi \sum_{n=1}^{\infty} \frac{q_n}{\rho^{m+1}} \sin[n\phi]$$

$\rho \to 1$

$$0 = 2\pi \cos[2\phi] - \pi \sum_{n=1}^{\infty} P_n \cos[n\phi] - \pi \sum_{n=1}^{\infty} q_n \sin[n\phi] \implies q_n = 0 \ , \ P_n = 0 \ (n \neq 2)$$

$$2\pi \cos[2\phi] - \pi P_2 \cos[2\phi] = 0 \quad , \quad P_2 = 2$$

$$t(1,\phi) = 2\cos[2\theta]$$

$$U(s,x) = \ln[R] - \sum_{m=1}^{\infty} \frac{1}{m} \left(\frac{\rho}{R}\right)^m \cos[m(\theta-\phi)] \quad , \quad \rho > R$$

$$\frac{\partial U}{\partial \rho} = L(s,x) = -\sum_{m=1}^{\infty} \frac{\rho^{m-1}}{R^m} \cos[m(\theta-\phi)] \quad , \quad \rho > R$$

$$\frac{\partial^2 U}{\partial R\, \partial \rho} = M(s,x) = \sum_{m=1}^{\infty} m \frac{\rho^{m-1}}{R^{m+1}} \cos[m(\theta-\phi)] \quad , \quad \rho > R$$

$$2\pi\, t(x) = \int_B M(s,x)\, u(s)\, dB(s) - \int_B L(s,x)\, t(s)\, dB(s)$$

$$2\pi\, t(x) = \int_0^{2\pi} \left[\sum_{m=1}^{\infty} m \frac{\rho^{m-1}}{R^{m+1}} \cos[m(\theta-\phi)]\right][\cos(2\theta)]\, d\theta +$$

$$\int_0^{2\pi} \left[\sum_{m=1}^{\infty} \frac{\rho^{m-1}}{R^{m}} \cos[m(\theta-\phi)]\right](2\cos[2\theta])\, d\theta$$

$(R = 1)$

$$= \int_0^{2\pi} \left[\sum_{m=1}^{\infty} m(\cos[m\theta]\cos[m\phi] + \sin[m\theta]\sin[m\phi])\right][\cos(2\theta)]\, d\theta$$

$$+ \int_0^{2\pi} \left[\sum_{m=1}^{\infty} \rho^{m-1}(\cos[m\theta]\cos[m\phi] + \sin[m\theta]\sin[m\phi])\right](2\cos[2\theta])\, d\theta$$

$=$

$$\int_0^{2\pi} \sum_{m=1}^{\infty} m\cos[m\theta]\cos[m\phi][\cos(2\theta)]\, d\theta + \int_0^{2\pi} \sum_{m=1}^{\infty} \rho^{m-1}\cos[m\theta]\cos[m\phi](2\cos[2\theta])\, d\theta$$

$$= \int_0^{2\pi} 2\rho\cos^2[2\theta]\cos[2\phi]\, d\theta + \int_0^{2\pi} 2\rho\cos^2[2\theta]\cos[2\phi]\, d\theta$$

$$= 2\rho\pi\cos[2\phi] + 2\rho\pi\cos[2\phi] = 4\rho\pi\cos[2\phi]$$

$$t(x) = 2\rho\cos[2\phi]$$

(2),

$$u(x) = \int_{B^+} U(s,x)\, \varphi(s)\, dB(s)$$

$$\text{set } \varphi(x) = P_0 + \sum_{n=1}^{\infty} (P_n\cos[n\theta] + q_n\sin[n\theta])$$

$u(x) =$

$$\int_{B^+} \left(\ln[R] - \sum_{m=1}^{\infty} \frac{1}{m}\left(\frac{\rho}{R}\right)^m \cos[m(\theta-\phi)]\right)\left(P_0 + \sum_{n=1}^{\infty}(P_n\cos[n\theta] + q_n\sin[n\theta])\right) dB(s)$$

$$= P_0\ln[R]\int_0^{2\pi} d\theta - \int_0^{2\pi}\sum_{m=1}^{\infty}\frac{1}{m} P_n\left(\frac{\rho}{R}\right)^m \cos[m\theta]\cos[m\phi]\cos[n\theta]\, d\theta -$$

$$\int_0^{2\pi}\sum_{m=1}^{\infty}\frac{1}{m} q_n\left(\frac{\rho}{R}\right)^m \sin[m\theta]\sin[m\phi]\sin[n\theta]\, d\theta$$

$R = 1$, $m = n$

$$u(x) = -\int_0^{2\pi}\sum_{n=1}^{\infty}\frac{1}{n} P_n\rho^n\cos^2[n\theta]\cos[n\phi]\, d\theta - \int_0^{2\pi}\sum_{n=1}^{\infty}\frac{1}{n} q_n\rho^n\sin^2[n\theta]\sin[n\phi]\, d\theta$$

$$= -\pi\sum_{n=1}^{\infty}\frac{1}{n} P_n\rho^n\cos[n\phi] - \pi\sum_{n=1}^{\infty}\frac{1}{n} q_n\rho^n\sin[n\phi]\, d\theta$$

$$u(1,\phi) = -\pi\sum_{n=1}^{\infty}\frac{1}{n} P_n\cos[n\phi] - \pi\sum_{n=1}^{\infty}\frac{1}{n} q_n\sin[n\phi]\, d\theta = \cos[2\phi]$$

$\Longrightarrow q_n = 0$, $P_n = 0$ $(n \neq 2)$

$\Longrightarrow n = 2$

$$-\pi\frac{1}{2}P_2\cos[2\phi] = \cos[2\phi] \Longrightarrow P_2 = \frac{-2}{\pi}$$

$$\varphi(s) = \frac{-2}{\pi}\cos[2\theta]$$

$$u(x) = \int_{B^+} U(s,x)\, \varphi(s)\, dB(s)$$

$$= \int_0^{2\pi} \left(\ln[R] - \sum_{m=1}^{\infty} \frac{1}{m} \left(\frac{\rho}{R}\right)^m \cos[m\,(\theta - \phi)] \right) \left(\frac{-2}{\pi} \cos[2\,\theta] \right) d\theta$$

$R = 1$

$$u\,(x) = \int_0^{2\pi} \left(-\sum_{m=1}^{\infty} \frac{1}{m}\, \rho^m \cos[m\,(\theta - \phi)] \right) \left(\frac{-2}{\pi} \cos[2\,\theta] \right) d\theta$$

$$= \int_0^{2\pi} \left(\frac{1}{\pi}\, \rho^2 \cos^2[2\,\theta] \cos[2\,\phi] \right) d\theta$$

$u\,(x) = \rho^2 \cos[2\,\phi]$