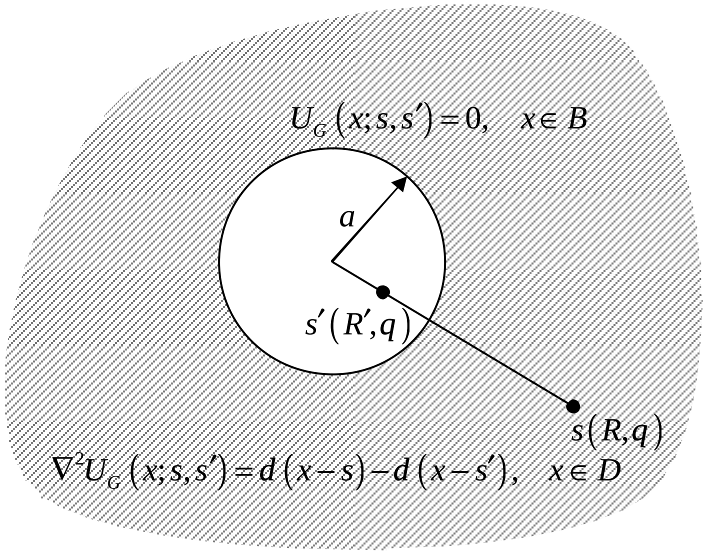

$$\nabla_x U_G(x,s)=2pd(x-s)\text{，}\; U_G(x,s)=0$$

$$\ln|x-s|=\ln R-\sum_{m=1}^{\infty}\frac{1}{m}\left(\frac{r}{R}\right)^m\cos m(q-f)\text{，}\quad R\geq r$$

$$\ln|x-s'|=\ln r-\sum_{m=1}^{\infty}\frac{1}{m}\left(\frac{R'}{r}\right)^m\cos m(q-f)\text{，}\quad r>R'$$

Choose $\dfrac{r}{R}=\dfrac{R'}{r}$, $R'=\dfrac{r^2}{R}=\dfrac{a^2}{R}$

$$\ln|x-s|-\ln|x-s'|=\ln R-\ln a$$

$$U_G(x,s)=\ln|x-s|-\ln|x-s'|-\ln R+\ln a$$

$$2pu(s)=\int T_G(x,s)\,u(x)\,dB(x)-\int U_G(x,s)\,t(x)\,dB(x)=\int T_G(x,s)\,u(x)\,dB(x)$$

$$T_G(x,s)=\frac{\partial\left(\ln|x-s|-\ln|x-s'|-\ln R+\ln a\right)}{\partial n_x}=\frac{r-\dfrac{R^2}{r}}{R^2+r^2-2Rr\cos(f-q)}$$

$$u(s)=\frac{1}{2p}\int_{2p}^{0}\frac{r-\dfrac{R^2}{r}}{R^2+r^2-2Rr\cos(f-q)}f(f)\,r\,df=\frac{1}{2p}\int_{2p}^{0}\frac{r^2-R^2}{R^2+r^2-2Rr\cos(f-q)}f(f)\,df$$

$$u(x)=\frac{1}{2p}\int_{0}^{2p}\frac{R^2-r^2}{R^2+r^2-2Rr\cos(f-q)}f(q)\,dq$$

$$\frac{2p}{r^2-R^2}u(x)=\int_{0}^{2p}\frac{f(q)}{R^2+r^2-2Rr\cos(f-q)}dq$$

$R=1$， $q=q$ ， $r=p$ ， $f=0$

$$u(x)=\frac{1}{2p}\int\frac{\left(p^2-1\right)f(q)}{1+p^2-2p\cos(q)}dq$$

1.

$f(q)=1$， $u(r,f)=1$

$$\int_0^{2p}\frac{1}{\left(p^2+1\right)-2p\cos q}dq=\frac{2p}{p^2-1}$$

2.

$f(q)=\cos 2q$ ， $u(r,f)=\frac{1}{r^2}\cos 2q$

$$\int_0^{2p}\frac{\cos 2q}{\left(p^2+1\right)-2p\cos q}dq=\frac{2p}{p^4-p^2}$$

3.

$$\int_0^{2p}\frac{p\cos q}{\left(p^2+1\right)-2p\cos q}dq=\frac{2p}{p^2-1}$$

$$\int_0^{2p}\frac{1}{\left(p^2+1\right)-2p\cos q}dq=\frac{2p}{p^2-1}$$

$$\int_0^{2p}\frac{1-p\cos q}{\left(p^2+1\right)-2p\cos q}dq=0$$

4.

$f(q)=\sin q$ ， $u(r,f)=\frac{1}{r}\sin q$

$$\int_0^{2p}\frac{\sin q}{\left(p^2+1\right)-2p\cos q}dq=0$$

5.

$$\int_0^{2p}\ln\sqrt{1+p^2-2p\cos q}\,dq=\int_0^{2p}\int_0^{p}\frac{r-r\cos q}{1+r^2-2r\cos q}dr\,dq=\int_0^{p}\int_0^{2p}\frac{r-r\cos q}{1+r^2-2r\cos q}dq\,dr$$

$f(q)=r$ ， $u(r,f)=r$

$$\int_0^p \int_0^{2p} \frac{r}{1 + r^2 - 2r\cos(q)} dq\, dr = \int_0^p \frac{2pr}{r^2 - 1} dr$$

$$f(q) = \cos q \text{ , } \; u(r, f) = \frac{1}{r} \cos f$$

$$\int_0^p \int_0^{2p} \frac{\cos q}{1 + r^2 - 2r\cos(q)} dq\, dr = \int_0^p \frac{2p}{\left(r^2 - 1\right) r} dr$$

$$\int_0^{2p} \ln \sqrt{1 + p^2 - 2p\cos(q)} dq = \int_0^p \frac{2pr}{r^2 - 1} dr - \int_0^p \frac{2p}{\left(r^2 - 1\right) r} dr$$

$$= \left[p \ln\left(r^2 - 1\right)\right]_0^p - 2p\left[-\ln r + \frac{1}{2}\ln\left(r^2 - 1\right)\right]_0^p$$

$$= 2p \ln p$$