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Short communication

A p-adaptive three dimensional boundary element method for elastostatic problems using quasi-Lagrange interpolation

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Abstract

An adaptive method for the determination of the order of element (or element order) was developed for the boundary element analysis of 3D elastostatic problems using quasi-Lagrange interpolation. Here the order of element means the highest order of polynomial function, which interpolates the displacement distribution in element. This method was based on acquiring the desired accuracy for each boundary element. From the numerical experiments, the relation $\xi = k(1/p)^{\beta}$ was deduced, where ξ is the error of the result of the boundary element analysis relative to the exact value, p is the order of element, and k and β are constants.

Applying this relation to the two results of computations with different orders of element, the order of element for the third computation was deduced. A computer program using this adaptive determination method for the order of element was developed and applied to several 3D elastostatic problems of various shapes. The usefulness of the method was illustrated by these application results.

Keywords: Computational mechanics; Numerical analysis; Boundary element method; Adaptive method; Adaptive p method; Accuracy estimation; Error analysis; Quasi-Lagrange element

1. Introduction

In the BEM analysis, and also in the FEM analysis, the approach to obtain analysis result of high accuracy by changing the discretization meshes or the order of elements based on the error analysis is known as the adaptive method. The adaptive method falls typically into three basic types: adaptive r method, adaptive h method and adaptive p method. Here the adaptive p method is called p-adaptive method. Among these adaptive methods, adaptive h method and adaptive p method are the main promising approaches when being used as independent method. Many research papers on the adaptive h method for BEM analysis have been published [1–9]. But few on the adaptive p method for BEM analysis have been reported [10–14].

In this paper, we attempt to propose an adaptive p method for BEM analysis with an accuracy guarantee technique—an approach of determination of the orders of elements (the orders of polynomial functions which interpolate the displacement distributions in elements) for the desired accuracy on the basis of error analysis. For this purpose, as presented in the following sections, at first, the relation between the order of element and the error of analysis was investigated based on some numerical experiments. Next, an algorithm of the adaptive method of determination of the order of element was deduced from the relation. Finally, a computer program accomplished with this method was developed and was applied to various kinds of problems to examine the usefulness of the method.

However, we cannot now draw a conclusion that the adaptive p method for BEM analysis proposed here is suitable for all kinds of geometry and boundary conditions, for example, the domain of crack tip which has a very small radius of curvature. This problem is still under study.

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2. Relation between the order of element and analysis error

In order to collect the fundamental information to establish our adaptive p method, the first step of the study was to investigate the relation between the order of element and analysis error by fundamental numerical experiments.

In the fundamental numerical experiments, the problems of a thick-walled spheroid subjected to an internal or external pressure were used. The exact solutions of the displacement and stress in these problems are known in the whole domain by an analytical method. In these numerical experiments, as shown in Fig. 1, the ratio *a*/*b* of the inner radius *a* to the outer radius *b* was assigned to 0, 0.2 and 0.5. Because of the symmetry, only the 1/8 domain was analyzed. The element division and the names assigned to the element groups are also shown in Fig. 1.

The boundary element adopted here was a curved triangular element with quasi-Lagrange interpolation functions instead of hierarchical interpolation functions [15] introduced by A. Peano. Because the order of quasi-Lagrange interpolation functions, here we call shape function, may vary from element to element without any loss of inter-element continuity conditions, it is especially suitable to be used in the adaptive p method.

Shape functions of order p, which have the maximum or nearly maximum values at the collocation points on the edges of triangular element are shown in Table 1. The positions of collocation points on the edges can be selected arbitrarily. In the study we adopted the position of these collocation points, the area coordinates of which are shown in Table 1.

Also shape functions of order p, which have the maximum or nearly maximum values at the collocation points in the inner domain of triangular element are shown in Table 2. The positions of collocation points in the inner domain can also be selected arbitrarily. In the study we adopted the position of these collocation points, the area coordinates of which are shown in Table 2.

Shape functions which have the maximum values at the three nodal points of triangular element are not shown, but they are linear functions of area coordinates such as ζ_i, ζ_j and ζ_k.

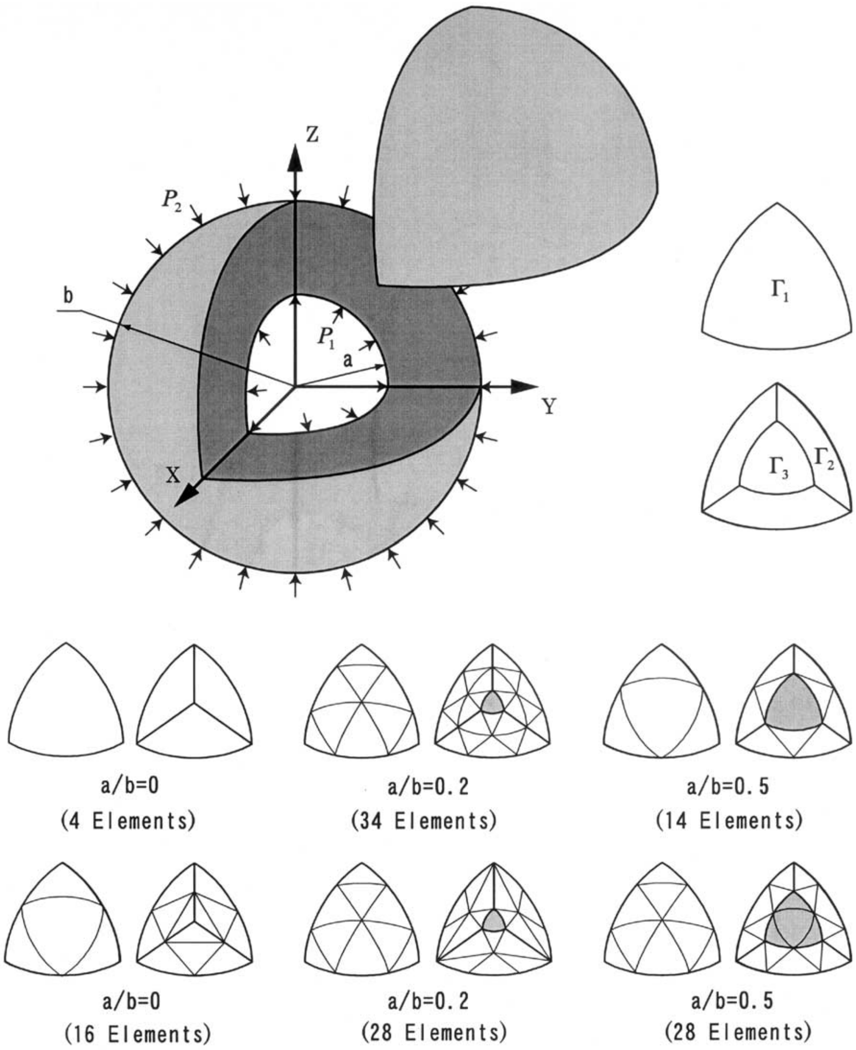

Fig. 1. Problem domain and mesh division for fundamental numerical experiments.

Table 1
Shape functions and area coordinates for collocation points on the edges of triangular element

p	Shape function	Area coordinate
2	$N_{2(1)} = (\zeta_i - 1)\zeta_i$	$(\zeta_i, \zeta_j, \zeta_k) = (\frac{1}{2}, \frac{1}{2}, 0)$
3	$N_{3(1-2)} = (\zeta_i - 1)(\zeta_i - \frac{1}{3})\zeta_i$	$(\zeta_i, \zeta_j, \zeta_k) = (\frac{2}{3}, \frac{1}{3}, 0)$
4	$N_{4(1-2)} = (\zeta_i - 1)(\zeta_i - \frac{1}{2})(\zeta_i - \frac{1}{4})\zeta_i$	$(\zeta_i, \zeta_j, \zeta_k) = (\frac{3}{4}, \frac{1}{4}, 0)$
	$N_{4(3)} = (\zeta_i - 1)(\zeta_i - \frac{3}{4})(\zeta_i - \frac{1}{4})\zeta_i$	$(\zeta_i, \zeta_j, \zeta_k) = (\frac{1}{2}, \frac{1}{2}, 0)$
5	$N_{5(1-2)} = (\zeta_i - 1)(\zeta_i - \frac{3}{5})(\zeta_i - \frac{2}{5})(\zeta_i - \frac{1}{5})\zeta_i$	$(\zeta_i, \zeta_j, \zeta_k) = (\frac{4}{5}, \frac{1}{5}, 0)$
	$N_{5(3-4)} = (\zeta_i - 1)(\zeta_i - \frac{4}{5})(\zeta_i - \frac{2}{5})(\zeta_i - \frac{1}{5})\zeta_i$	$(\zeta_i, \zeta_j, \zeta_k) = (\frac{3}{5}, \frac{2}{5}, 0)$
6	$N_{6(1-2)} = (\zeta_i - 1)(\zeta_i - \frac{2}{3})(\zeta_i - \frac{1}{2})(\zeta_i - \frac{1}{3})(\zeta_i - \frac{1}{6})\zeta_i$	$(\zeta_i, \zeta_j, \zeta_k) = (\frac{5}{6}, \frac{1}{6}, 0)$
	$N_{6(3-4)} = (\zeta_i - 1)(\zeta_i - \frac{5}{6})(\zeta_i - \frac{1}{2})(\zeta_i - \frac{1}{3})(\zeta_i - \frac{1}{6})\zeta_i$	$(\zeta_i, \zeta_j, \zeta_k) = (\frac{2}{3}, \frac{1}{3}, 0)$
	$N_{6(5)} = (\zeta_i - 1)(\zeta_i - \frac{5}{6})(\zeta_i - \frac{2}{3})(\zeta_i - \frac{1}{3})(\zeta_i - \frac{1}{6})\zeta_i$	$(\zeta_i, \zeta_j, \zeta_k) = (\frac{1}{2}, \frac{1}{2}, 0)$

The relation to be obtained is the relation between the order of element and analysis error of the displacement and of the traction in the whole domain of the problem. This relation is indispensable for the adaptive determination of the order of each element for the third meshing in our adaptive p method.

The results of the fundamental numerical experiments are shown in Figs. 2–5. These figures show the relation between the reciprocal of the order of element and the error parameters of the analyses, which are defined by

$$\xi = \frac{\int_\Gamma (u_{\mathrm{BEM}} - u_{\mathrm{E}})^2 \mathrm{d}\Gamma}{\int_\Gamma (u_{\mathrm{E}})^2 \mathrm{d}\Gamma} \tag{1}$$

and

$$\xi = \frac{\int_\Gamma (t_{\mathrm{BEM}} - t_{\mathrm{E}})^2 \mathrm{d}\Gamma}{\int_\Gamma (t_{\mathrm{E}})^2 \mathrm{d}\Gamma} \tag{2}$$

corresponding to the displacement and the stress, respectively. Here u_{BEM} and t_{BEM} denote the BEM solutions, while u_{E} and t_{E} denote the exact solutions. The different loads, mesh divisions and element groups are distinguished by the annotations in these figures. IP and OP denote the internal pressure and the external pressure, respectively. $4E \sim 34E$ indicate the number of elements, and $\Gamma_1 \sim \Gamma_3$ indicate the names of the groups of elements shown in Fig. 1.

From these figures we can see the fact that the lines of the relation between the reciprocal of the order p of element and the error parameter ξ of the displacement or of the traction can be regarded approximately as straight lines. Based on these convergence characteristics, the relations in the form of

$$\frac{\int_\Gamma (u_{\mathrm{BEM}} - u_{\mathrm{E}})^2 \mathrm{d}\Gamma}{\int_\Gamma (u_{\mathrm{E}})^2 \mathrm{d}\Gamma} = k\left(\frac{1}{p}\right)^\beta \tag{3}$$

and

$$\frac{\int_\Gamma (t_{\mathrm{BEM}} - t_{\mathrm{E}})^2 \mathrm{d}\Gamma}{\int_\Gamma (t_{\mathrm{E}})^2 \mathrm{d}\Gamma} = k\left(\frac{1}{p}\right)^\beta \tag{4}$$

Table 2
Shape functions and area coordinates for collocation points in the inner domain of triangular element

p	Shape function	Area coordinate
3	$N_3 = \zeta_i\zeta_j\zeta_k$	$(\zeta_i, \zeta_j, \zeta_k) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$
4	$N_{4(1-3)} = \zeta_i\zeta_j\zeta_k(\zeta_i - \frac{1}{4})$	$(\zeta_i, \zeta_j, \zeta_k) = (\frac{1}{2}, \frac{1}{4}, \frac{1}{4})$
5	$N_{5(1-3)} = \zeta_i\zeta_j\zeta_k(\zeta_i - \frac{1}{5})(\zeta_i - \frac{2}{5})$	$(\zeta_i, \zeta_j, \zeta_k) = (\frac{3}{5}, \frac{1}{5}, \frac{1}{5})$
	$N_{5(4-6)} = \zeta_i\zeta_j\zeta_k(\zeta_j - \frac{1}{5})(\zeta_k - \frac{1}{5})$	$(\zeta_i, \zeta_j, \zeta_k) = (\frac{1}{5}, \frac{2}{5}, \frac{2}{5})$
6	$N_{5(1-3)} = \zeta_i\zeta_j\zeta_k(\zeta_i - \frac{1}{6})(\zeta_i - \frac{1}{3})(\zeta_i - \frac{1}{2})$	$(\zeta_i, \zeta_j, \zeta_k) = (\frac{2}{3}, \frac{1}{6}, \frac{1}{6})$
	$N_{5(4-6)} = \zeta_i\zeta_j\zeta_k(\zeta_j - \frac{1}{6})(\zeta_j - \frac{1}{3})(\zeta_k - \frac{1}{6})$	$(\zeta_i, \zeta_j, \zeta_k) = (\frac{1}{6}, \frac{1}{2}, \frac{1}{3})$
	$N_{5(7-9)} = \zeta_i\zeta_j\zeta_k(\zeta_k - \frac{1}{6})(\zeta_k - \frac{1}{3})(\zeta_j - \frac{1}{6})$	$(\zeta_i, \zeta_j, \zeta_k) = (\frac{1}{6}, \frac{1}{3}, \frac{1}{2})$
	$N_{5(10)} = \zeta_i\zeta_j\zeta_k(\zeta_i - \frac{1}{6})(\zeta_j - \frac{1}{6})(\zeta_k - \frac{1}{6})$	$(\zeta_i, \zeta_j, \zeta_k) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$

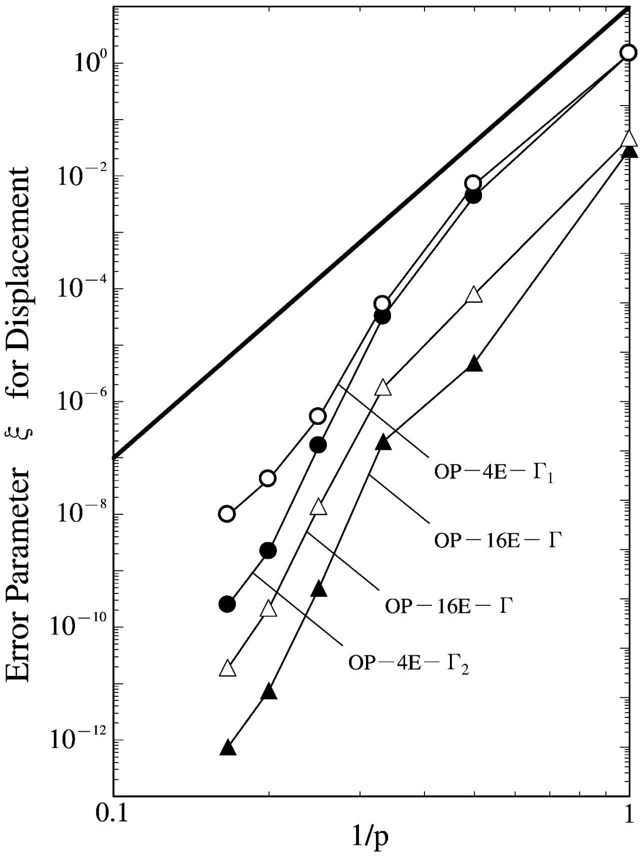

Fig. 2. Convergence characteristics for displacement ($a/b = 0$).

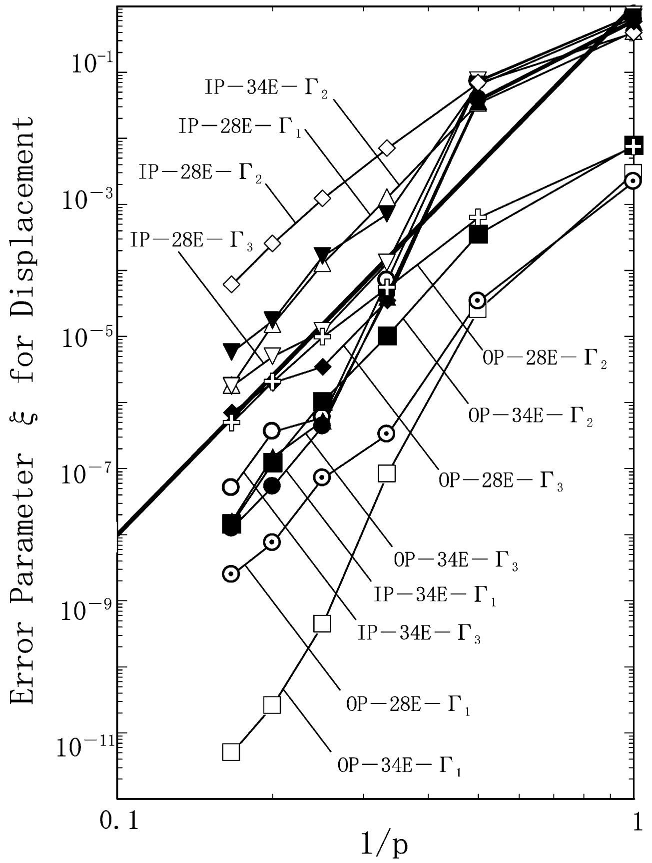

Fig. 3. Convergence characteristics for displacement ($a/b = 0.2$).

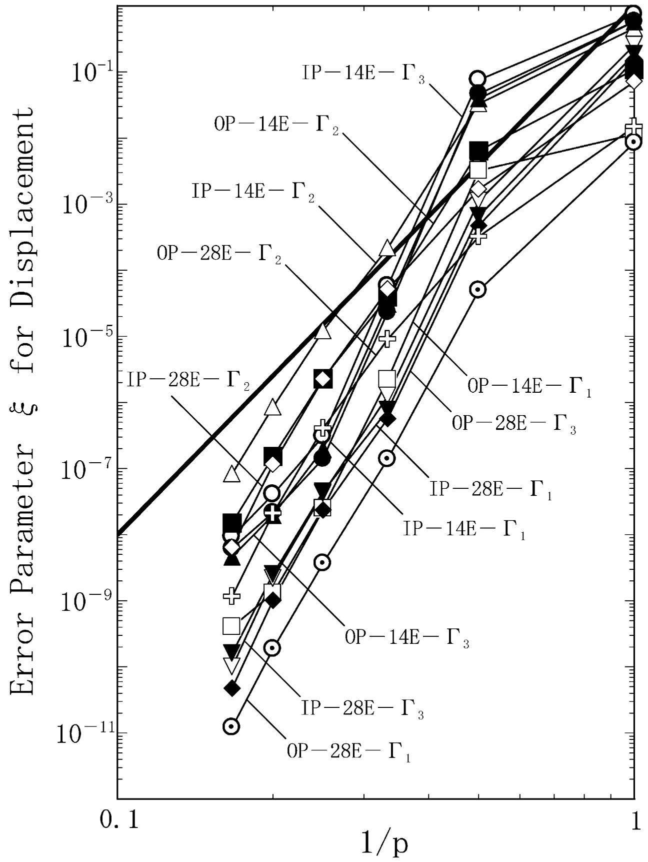

Fig. 4. Convergence characteristics for displacement ($a/b = 0.5$).

were obtained. The value of β, which denotes the average slope of the lines was estimated to be 8 in the case of displacement and to be 6 in the case of stress from the slopes of the straight lines, respectively, drawn on the logarithmic graphs. And k is constant.

3. Adaptive determination of the order of element

In this chapter, we propose a method for the adaptive determination of the orders of elements for the adaptive p method for BEM analysis using Eqs. (3) or (4).

From the numerical experiments using the thick-walled spheroid subjected to an internal or external pressure, the relations between the reciprocal of the order p of element and the error parameter ξ of the displacement or of the traction were found to be approximately straight lines on the logarithmic graphs, hence Eqs. (3) and (4) were obtained. Although there is no theory, which could be applied to prove them and it is not yet known whether they are suitable for other problems, we consider these relations as tenable ones and use these relations to establish a method of determination of the order of element.

In the following, a method of the adaptive determination of the order of element in the case of displacement is established. A method for the traction is omitted, because the process to establish this method is the same to the process for the traction.

Eq. (3) can be applied to two stages of computations, which are performed one after another. At the $(\nu-1)$th stage of computation, due to Eq. (3), the relation between the order p of element and the error parameter ξ of displacement is given by

$$\frac{\int_{\Gamma}(u_{\mathrm{BEM}}^{(\nu-1)}-u_{\mathrm{E}})^2\mathrm{d}\Gamma}{\int_{\Gamma}(u_{\mathrm{E}})^2\mathrm{d}\Gamma}=k\left(\frac{1}{p^{(\nu-1)}}\right)^{\beta} \tag{5}$$

Eq. (3) is applied to every group of elements. In the case of the succeeding (ν)th stage of computation, the relation between the order p of element and the error parameter ξ of traction is similarly given by

$$\frac{\int_{\Gamma}(u_{\mathrm{BEM}}^{(\nu)}-u_{\mathrm{E}})^2\mathrm{d}\Gamma}{\int_{\Gamma}(u_{\mathrm{E}})^2\mathrm{d}\Gamma}=k\left(\frac{1}{p^{(\nu)}}\right)^{\beta} \tag{6}$$

Subtracting Eq. (6) from Eq. (5), we can get

$$\frac{\int_{\Gamma}\{(u_{\mathrm{BEM}}^{(\nu-1)}-u_{\mathrm{E}})^2-(u_{\mathrm{BEM}}^{(\nu)}-u_{\mathrm{E}})^2\}\mathrm{d}\Gamma}{\int_{\Gamma}(u_{\mathrm{E}})^2\mathrm{d}\Gamma}$$

$$=k\left\{\left(\frac{1}{p^{(\nu-1)}}\right)^{\beta}-\left(\frac{1}{p^{(\nu)}}\right)^{\beta}\right\} \tag{7}$$

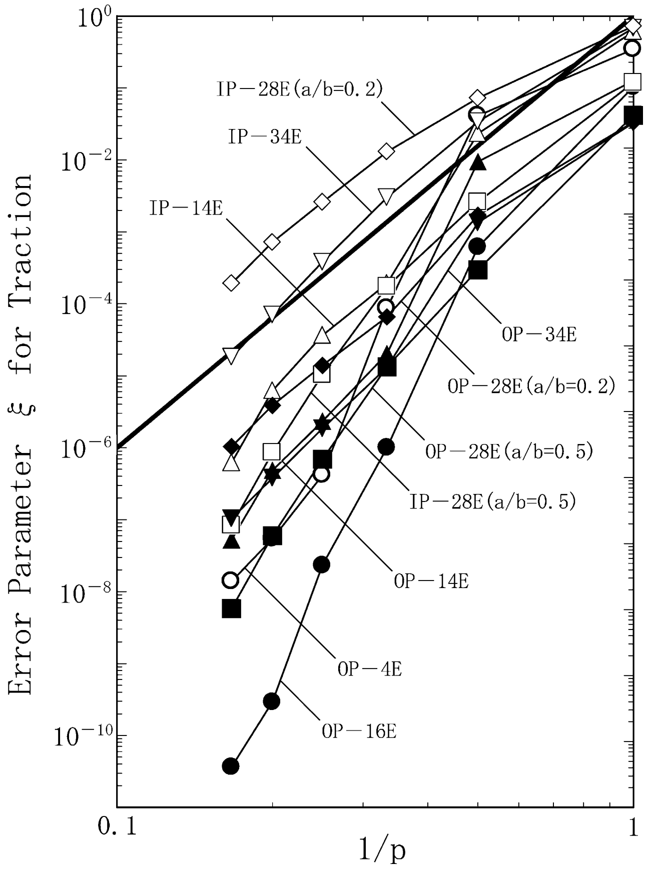

Fig. 5. Convergence characteristics for traction.

Provided that the fully converged displacement is either monotonically decreasing or monotonically increasing, we have

$$(u_{\mathrm{BEM}}^{(\nu-1)} - u_{\mathrm{E}})^2 - (u_{\mathrm{BEM}}^{(\nu)} - u_{\mathrm{E}})^2 \geq (u_{\mathrm{BEM}}^{(\nu-1)} - u_{\mathrm{BEM}}^{(\nu)})^2 \tag{8}$$

Hence, Eq. (7) can be written into an inequality equation as

$$k\left\{\left(\frac{1}{p^{(\nu-1)}}\right)^{\beta} - \left(\frac{1}{p^{(\nu)}}\right)^{\beta}\right\} \geq \frac{\int_{\Gamma}(u_{\mathrm{BEM}}^{(\nu-1)} - u_{\mathrm{BEM}}^{(\nu)})^2 \mathrm{d}\Gamma}{\int_{\Gamma}(u_{\mathrm{E}})^2 \mathrm{d}\Gamma} \tag{9}$$

Solving for k, we can get

$$k \geq \frac{\int_{\Gamma}(u_{\mathrm{BEM}}^{(\nu-1)} - u_{\mathrm{BEM}}^{(\nu)})^2 \mathrm{d}\Gamma}{\left\{\left(\frac{1}{p^{(\nu-1)}}\right)^{\beta} - \left(\frac{1}{p^{(\nu)}}\right)^{\beta}\right\}\int_{\Gamma}(u_{\mathrm{E}})^2 \mathrm{d}\Gamma} \tag{10}$$

If the final stage of computation, i.e. $(\nu+1)$th stage of computation, gives the solution $u_{\mathrm{BEM}}^{(\nu+1)}$ with an error smaller than the allowable error θ, we can write

$$\theta \geq \left|\frac{u_{\mathrm{BEM}}^{(\nu+1)} - u_{\mathrm{E}}}{u_{\mathrm{E}}}\right| \tag{11}$$

Using this and the relation between the order $p^{(\nu+1)}$ of element and displacement $u_{\mathrm{BEM}}^{(\nu+1)}$ at the $(\nu+1)$th stage of

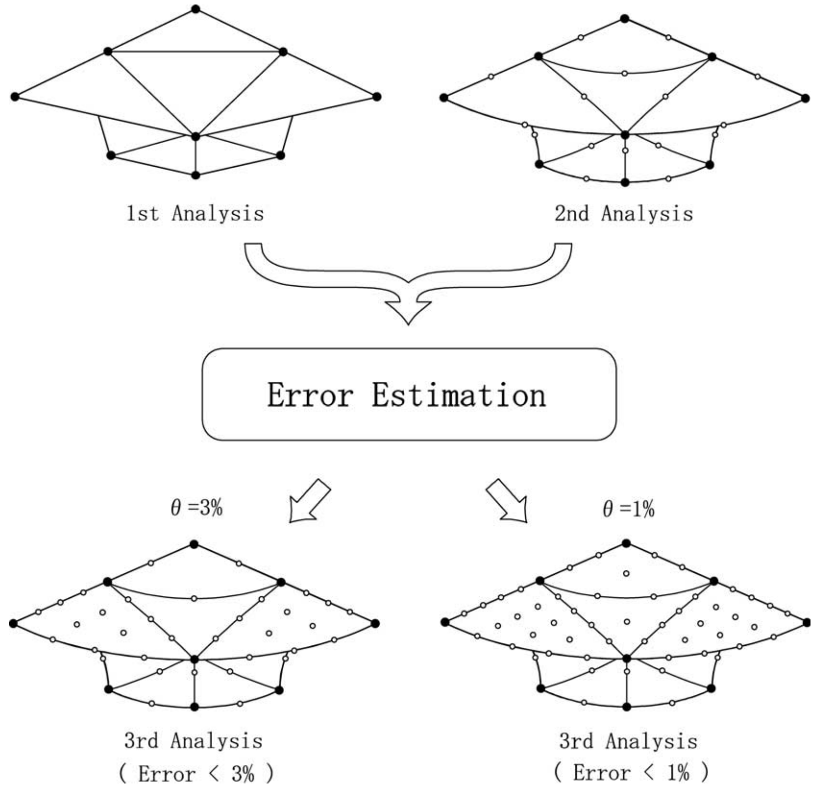

Fig. 6. Process of p-adaptive boundary element method.

computation, we can further get

$$\theta^2 \geq \frac{\int_\Gamma (u_{\mathrm{BEM}}^{(\nu+1)} - u_{\mathrm{E}})^2 \mathrm{d}\Gamma}{\int_\Gamma (u_{\mathrm{E}})^2 \mathrm{d}\Gamma} = k\left(\frac{1}{p^{(\nu+1)}}\right)^\beta \tag{12}$$

Combining the inequality Eq. (10) with Eq. (12), and solving for $p^{(\nu+1)}$, we have

$$(p^{(\nu+1)})^\beta \geq \frac{\int_\Gamma (u_{\mathrm{BEM}}^{(\nu-1)} - u_{\mathrm{BEM}}^{(\nu)})^2 \mathrm{d}\Gamma}{\theta^2 \left\{ \left(\frac{1}{p^{(\nu-1)}}\right)^\beta - \left(\frac{1}{p^{(\nu)}}\right)^\beta \right\} \int_\Gamma (u_{\mathrm{E}})^2 \mathrm{d}\Gamma} \tag{13}$$

In the state of fully converged displacement, $u_{\mathrm{BEM}}^{(\nu)}$ is the best estimate to u_{E}. So we can have

$$(p^{(\nu+1)})^\beta \geq \frac{\int_\Gamma (u_{\mathrm{BEM}}^{(\nu-1)} - u_{\mathrm{BEM}}^{(\nu)})^2 \mathrm{d}\Gamma}{\theta^2 \left\{ \left(\frac{1}{p^{(\nu-1)}}\right)^\beta - \left(\frac{1}{p^{(\nu)}}\right)^\beta \right\} \int_\Gamma (u_{\mathrm{BEM}}^{(\nu)})^2 \mathrm{d}\Gamma} \tag{14}$$

Based on this inequality equation, the necessary order p of element which makes the error of displacement satisfy the allowable error θ could be found. This order p of element is an average order of all elements in every group of elements.

Considering the error of each element on the basis of the average error of a group of elements, we can get the equation

$$(p_i^{(\nu+1)})^\beta \geq \frac{\int_\Gamma (u_{\mathrm{BEM}i}^{(\nu-1)} - u_{\mathrm{BEM}i}^{(\nu)})^2 \mathrm{d}\Gamma}{\theta^2 \left\{ \left(\frac{1}{p_i^{(\nu-1)}}\right)^\beta - \left(\frac{1}{p_i^{(\nu)}}\right)^\beta \right\} \int_\Gamma (u_{\mathrm{BEM}i}^{(\nu)})^2 \mathrm{d}\Gamma} \tag{15}$$

by which the order $p_i^{(\nu+1)}$ of ith element at the $(\nu+1)$th stage of computation can be computed. By applying Eq. (15) to every element of the whole domain, an appropriate global approximation to give a desired accuracy is expectable.

By similar deducing procedure, we can alternatively get the equation

$$(p_i^{(\nu+1)})^\beta \geq \frac{\int_\Gamma (t_{\mathrm{BEM}i}^{(\nu-1)} - t_{\mathrm{BEM}i}^{(\nu)})^2 \mathrm{d}\Gamma}{\theta^2 \left\{ \left(\frac{1}{p_i^{(\nu-1)}}\right)^\beta - \left(\frac{1}{p_i^{(\nu)}}\right)^\beta \right\} \int_\Gamma (t_{\mathrm{BEM}i}^{(\nu)})^2 \mathrm{d}\Gamma} \tag{16}$$

for traction.

When the order of every element calculated by Eq. (15) or (16) is used, it is to be expected that the result of BEM will satisfy the allowable error.

Because the orders of elements are different from each other, the orders on the element sides need to be adjusted to keep the continuity between the neighboring elements. The order on a side of element is assigned to the higher order of two neighboring elements.

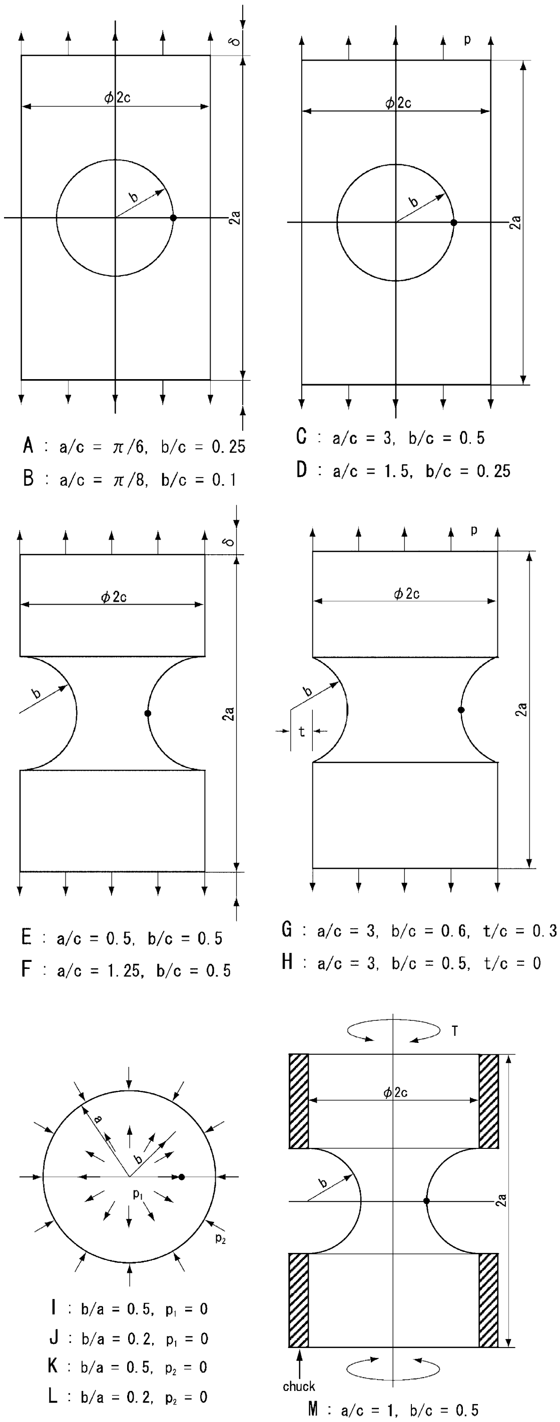

Fig. 7. Problem used for numerical experiments.

On the basis of these equations and explanation, the error estimation or accuracy guarantee of the adaptive p method for BEM analysis was carried out according to the flow shown in Fig. 6. At the first stage of computation, the linear element was used, and at the second stage, the quadratic element was used. At the third stage, the order of element calculated by Eq. (15) or (16) was used. The value of the allowable error θ was assigned to, for example, 3 and 1%.

Table 3
Results of the numerical experiments

Problem	3%		1%		Exact value
	BEM	Error (%)	BEM	Error (%)	
A	1.991	1.066	1.953	0.863	1.970
B	2.060	1.879	2.008	0.692	2.022
C	1.759	1.512	1.791	0.280	1.786
D	1.925	1.383	1.959	0.359	1.952
E	1.384	2.672	1.401	1.477	1.422
F	1.375	2.758	1.387	1.909	1.414
G	1.446	3.856	1.488	1.064	1.504
H	1.381	2.126	1.406	0.354	1.411
I	1.755	2.392	1.715	0.058	1.714
J	1.471	2.711	1.520	0.529	1.512
K	0.734	2.801	0.709	0.700	0.714
L	0.522	1.953	0.518	1.172	0.512
M	1.179	2.700	1.139	0.784	1.148

4. Results of numerical experiments

A computer program using the adaptive p method for the accuracy guarantee mentioned above was developed and applied to various 3D elastostatic problems to examine the usefulness of our adaptive p method.

As mentioned before, at the first stage of computation, the linear element was used, and at the second stage, the quadratic element was used. At the third stage of computation, the order of element calculated by Eq. (16) was used. The value of the allowable error θ was assigned to be 3 and 1%.

For the examination of the usefulness of our adaptive p method, the program was applied to 13 problems shown in Fig. 7. The stress concentration factors of these problems were computed. For the problems (A) $\sim$ (H), and (M), the precise numerical solutions by 3D theory of elasticity

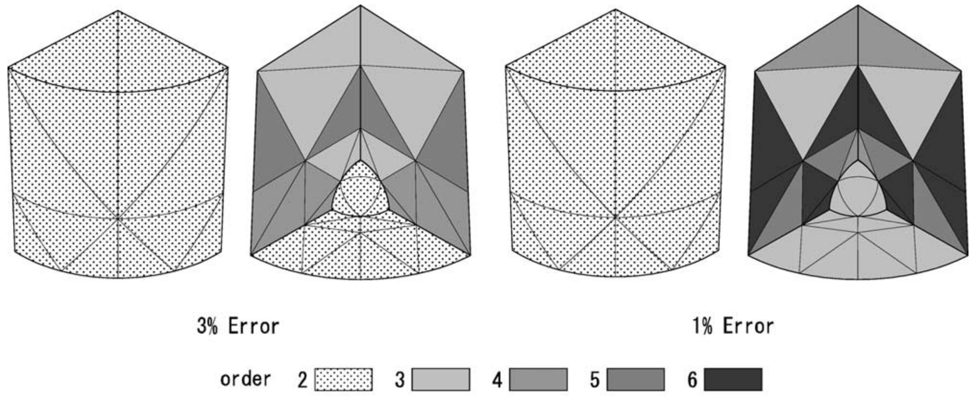

Fig. 8. Order of elements for the third computation (problem D).

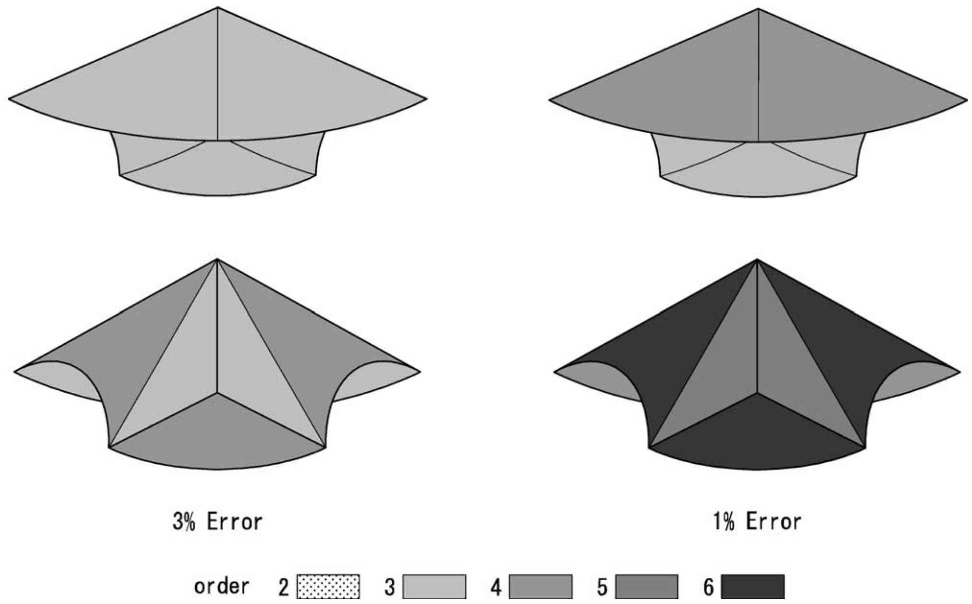

Fig. 9. Order of elements for the third computation (problem E).

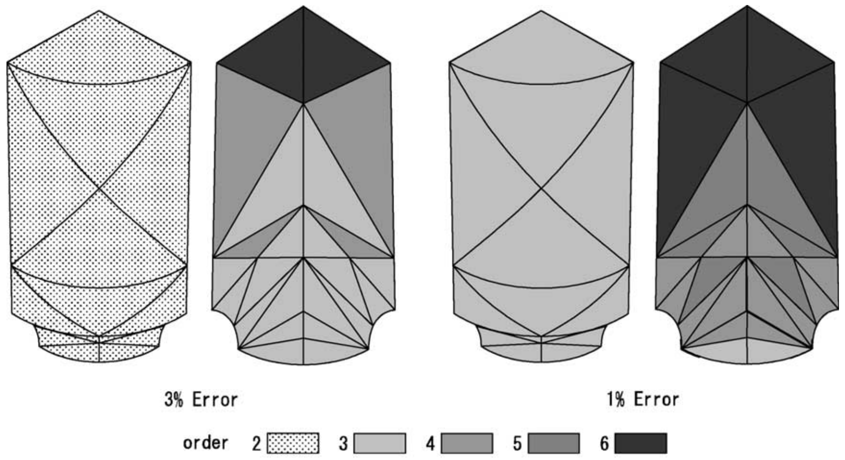

Fig. 10. Order of elements for the third computation (problem G).

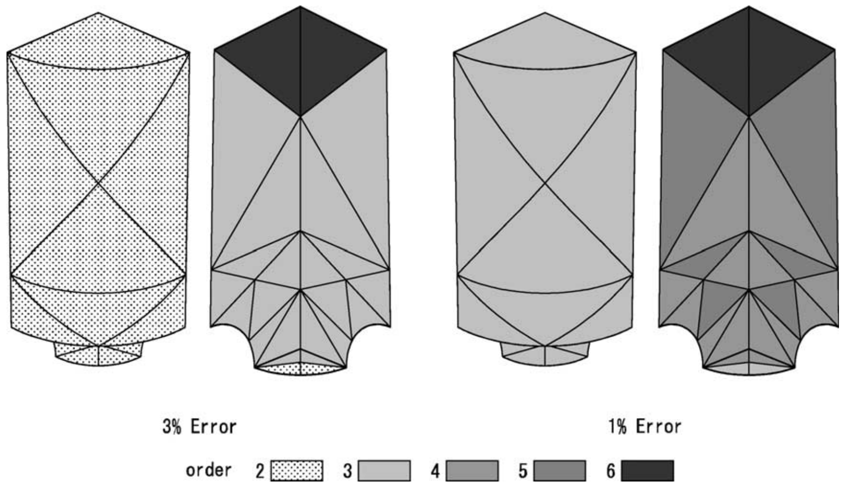

Fig. 11. Order of elements for the third computation (problem H).

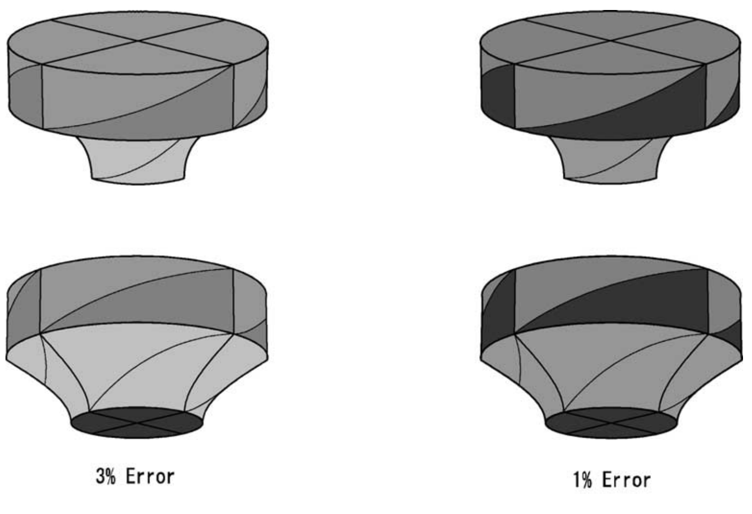

Fig. 12. Order of elements for the third computation (problem M).

[16–19] and the solutions [20] for the cylinders with an infinite length obtained by the Body Force Method, etc. are used as exact solutions. The exact analytical solutions for the problems (I) ~ (L) are known. The black points in the figures denote the maximum stress points to which attention was paid. Due to the symmetry, only the 1/8 domain for problems (A) ~ (H), and (M), and the 1/2 domain for problems (I) ~ (L) were analyzed. The stress concentration factor at the black point was the averaged value of the stress concentration factors computed from elements neighboring the black point.

The result of the numerical experiments is given in Table 3. The error (Error) in % is the error of the BEM solution (BEM) relative to the exact value (Exact value).

According to Table 3, although there are a few error values exceeding the allowable error θ, most of them are lower than the allowable level or around it nearly. From this fact, the method of the accuracy guarantee method proposed here can be considered as a feasible one.

Figs. 8–12 show the orders of elements for the third computations for problems D, E, G, H, and M, respectively.

5. Concluding remarks

To establish an accuracy guarantee or error estimation method for the adaptive p method for 3D BEM, the relations between the order of element and the analysis error was first investigated by the fundamental numerical experiments using thick-walled spheroids subjected to the internal or external pressure.

On the basis of these equations, a method of the adaptive determination of the order of element was devised, where BEM analyses were performed twice with different orders of elements and the necessary order of each element was determined based on the preceding computations. The result computed with these orders is expected to satisfy the allowable error.

Finally, an adaptive analysis program using this method was developed and applied to the analysis of various 3D elastostatic problems. The usefulness of this method was illustrated by these application results.

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