

The SDOF second ODE

$$y''(t) + 2\xi\omega y'(t) + \omega^2 y(t) = 0, \quad y(0) = y_0, \quad y'(0) = v_0$$

Integrating and substituting the initial conditions

$$y'(t) - v_0 + 2\xi\omega [y(t) - y_0] + \omega^2 \int_0^t y(t_1) dt_1 = 0$$

$$y'(t) + 2\xi\omega y(t) + \omega^2 \int_0^t y(t_1) dt_1 = v_0 + 2\xi\omega y_0$$

Integrating and substituting the initial conditions

$$y(t) - y_0 + 2\xi\omega \int_0^t y(t_2) dt_2 + \omega^2 \int_0^t \int_0^{t_1} y(t_2) dt_2 dt_1 = (v_0 + 2\xi\omega y_0)t$$

$$\text{Using } \int_a^x \int_a^{x_n} \int_a^{x_{n-1}} \cdots \int_a^{x_3} \int_a^{x_2} f(x_1) dx_1 dx_2 dx_3 \cdots dx_n = \frac{1}{(n-1)!} \int_a^x (x-\zeta)^{n-1} f(\zeta) d\zeta$$

And we can get

$$y(t) + 2\xi\omega \int_0^t y(t_3) dt_3 + \omega^2 \int_0^t (t-t_3) y(t_3) dt_3 = (v_0 + 2\xi\omega y_0)t + y_0$$

$$y(t) = \int_0^t -[2\xi\omega + \omega^2(t-s)] y(s) ds + (v_0 + 2\xi\omega y_0)t + y_0$$

$$\therefore K(t,s) = -[2\xi\omega + \omega^2(t-s)], \quad g(t) = (v_0 + 2\xi\omega y_0)t + y_0$$

It' s second kind of Volterra form.

It' s not symmetry.

It' s regular.