Journal of Sound and Vibration (1995) **187**(5), 781–798

ANALYSIS OF THE NEAR FIELD ACOUSTIC RADIATION CHARACTERISTICS OF TWO RADIALLY VIBRATING SPHERES BY THE HELMHOLTZ INTEGRAL EQUATION FORMULATION AND A CRITICAL STUDY OF THE EFFICACY OF THE "CHIEF" OVERDETERMINATION METHOD IN TWO-BODY PROBLEMS

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(*Received* 22 *May* 1991, *and in final form* 30 *September* 1994)

The formulation of acoustic radiation from multiple vibrating bodies of arbitrary shape by the Helmholtz integral equation is presented. A computer code was developed to calculate the sound pressure field around an arbitrary number of three-dimensional vibrating bodies of arbitrary shape. The near field of two dilating spheres, determined by using this code, is presented in the form of equal pressure contours. The paper also presents a critical study of the efficacy of the CHIEF method in acoustic radiation problems involving more than one vibrating body.

1. INTRODUCTION

This paper is concerned with the computation of the acoustic pressure field around an arbitrary number of vibrating non-compact sources, having arbitrary shapes, embedded in a homogeneous medium which is initially at rest. The analysis is based on the numerical implementation of the exterior surface Helmholtz integral equation, SHIE, by using the isoparametric boundary element technique. This method has been used extensively in the numerical determination of acoustic radiation from an arbitrarily shaped single body. It is also applicable to radiation and/or scattering problems involving more than one body, and several papers have dealt with this case [1–6].

A disadvantage of SHIE is that it can yield only a non-unique solution when the forcing frequency equals one of the characteristic frequencies associated with the interior of the body [7]. Of the variety of methods proposed to eliminate the impact of this on the numerical results, the method of Schenck, CHIEF [7], and the method of Burton and Miller [8] appear to compete. However, as far as the present authors are aware, no paper has been published to date reporting on the efficacy of these methods in radiation problems involving more than one vibrating body. It is the purpose of this paper to study the efficacy of the SHIE and CHIEF methods in multi-body radiation problems, in the vicinity of the critical frequencies, and to present data on the near field characteristics of two dilating spheres. The study was

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motivated by the need to predict the near field of a number of non-compact noise sources operating in free field conditions.

The numerical formulation is similar to that given in reference [9] except that the computer code developed can cater for the presence of a number of disjointed closed surfaces in three dimensions. Numerical results are presented in this paper, for simplicity, only for two dilating spheres, although the main advantage of the methods considered lies in their versatility in dealing with arbitrarily shaped bodies. The paper includes a discussion of the effect of frequency on the near field equal pressure contours as well as the impact of the non-uniqueness problem and the extent to which it can be remedied by the CHIEF method.

2. HELMHOLTZ INTEGRAL EQUATION FOR MULTIPLE BODIES

For a vibrating body with a boundary S and a unit outward normal $\mathbf{n}$, classical Helmholtz integral equation may be written in terms of the sound pressure amplitude p and the normal velocity amplitude u, as

$$C(\mathbf{x})p(\mathbf{x}) = \int_S \{p(\mathbf{y})[\partial G(R, k)/\partial n(\mathbf{y})] + \mathrm{i}z_0 k u(\mathbf{y}) G(R, k)\}\, \mathrm{d}s(\mathbf{y}), \tag{1}$$

where $\mathbf{y}$ is any point on S, $\mathbf{x}$ is any point in space, $R = |\mathbf{x} - \mathbf{y}|$, $\mathrm{i} = \sqrt{(-1)}$. k denotes the wavenumber $2\pi f/c$ where f is the frequency, c is the speed of sound and $\exp(\mathrm{i}2\pi f t)$ time dependence is assumed, $\mathrm{d}s$ denotes a differential boundary element, $G(R, k)$ is the free space Green function $G = \exp(-\mathrm{i}kR)/R$ for the Helmholtz operator in three dimensions, and z_0 is the characteristic impedance of the medium $z_0 = \rho_0 c$ where ρ_0 is the density of the medium at rest. For $\mathbf{x}$ in the exterior, $C(\mathbf{x})$ is equal to 4π; for $\mathbf{x}$ in the interior, $C(\mathbf{x}) = 0$ and for $\mathbf{x}$ on S, $C(\mathbf{x})$ is given by [9]

$$C(\mathbf{x}) = 4\pi + \int_S [\partial(1/R)/\partial n(\mathbf{y})]\, \mathrm{d}s(\mathbf{y}). \tag{2}$$

If there are two or more vibrating bodies embedded in the acoustic medium, equation (1) can be applied by taking $S = S_1 \cup S_2 \cup \cdots \cup S_B$ where S_i ($i = 1, 2, \ldots, B$) denotes the boundary of the ith body and B the total number of bodies.

For $\mathbf{x}$ on S, equation (1) is called the exterior surface Helmholtz integral equation (SHIE). The numerical implementation of SHIE by the boundary element method gives a set of complex algebraic equations which may be written as [10]

$$[\mathbf{E} - \mathbf{K}(f)]\mathbf{p} = \mathbf{H}(f)\mathbf{u}. \tag{3}$$

Here $\mathbf{p}$ and $\mathbf{u}$ are the nodal sound pressure and the prescribed nodal normal velocity vectors, respectively. Matrices $\mathbf{E}$ (a diagonal matrix), $\mathbf{K}(f)$ and $\mathbf{H}(f)$ are of size $N \times N$, where N is

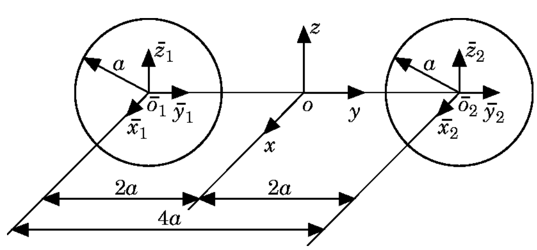

Figure 1. Two spherical source model.

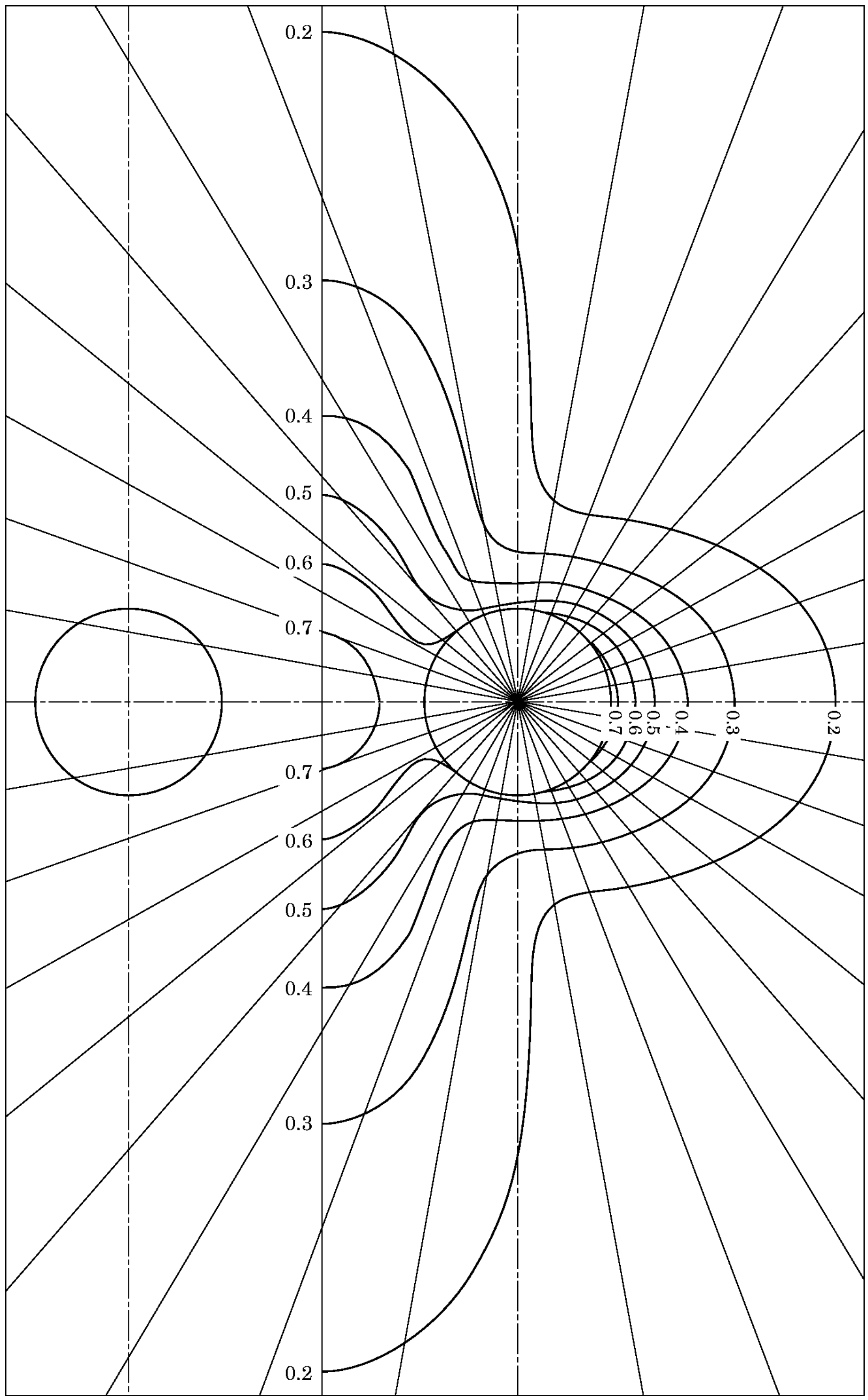

Figure 2. Equal pressure contours of two dilating spherical sources at the $z = 0$ plane for $ka = 1$.

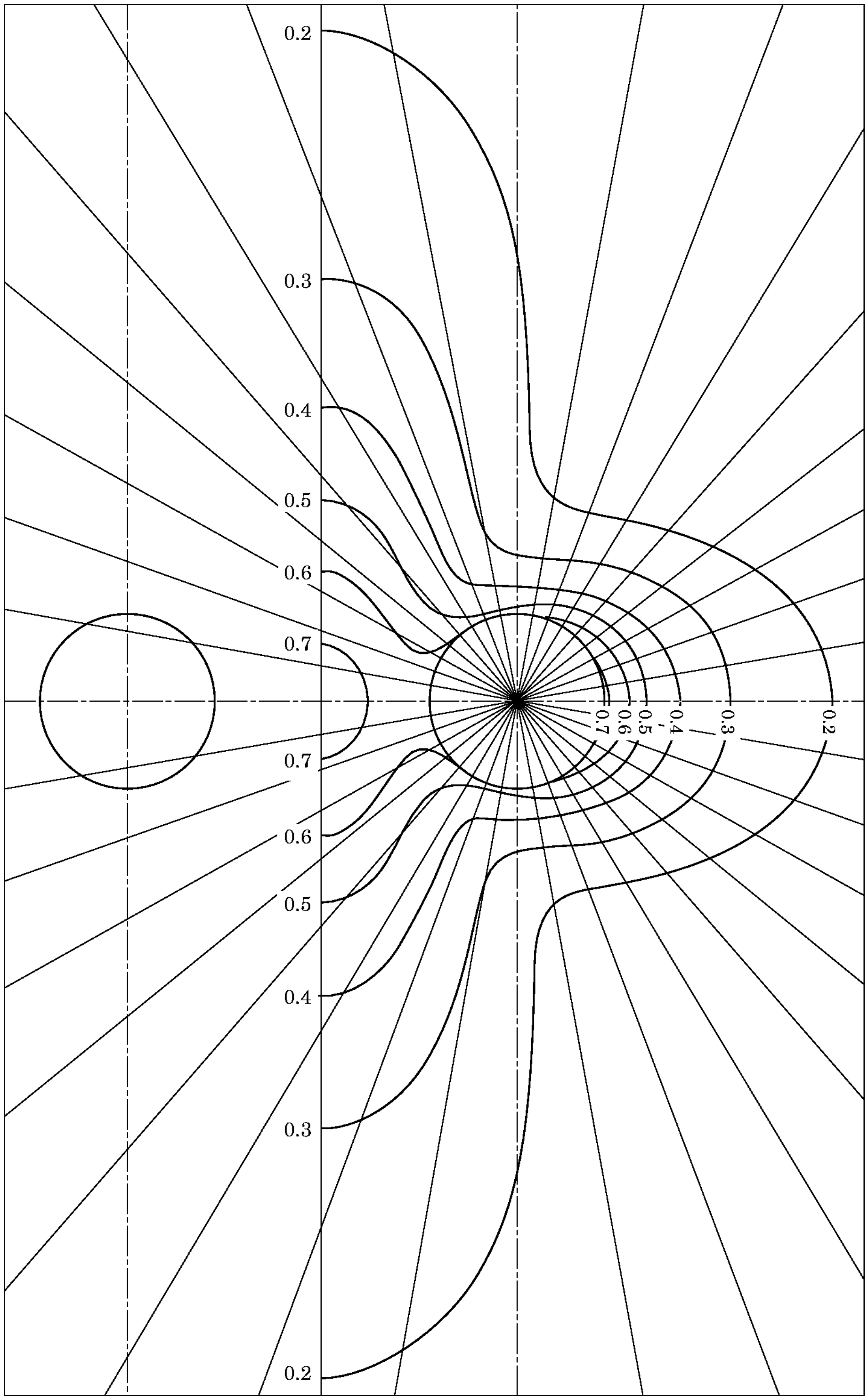

Figure 3. Equal pressure contours of two dilating spherical sources at the $z = a/3$ plane for $ka = 1$.

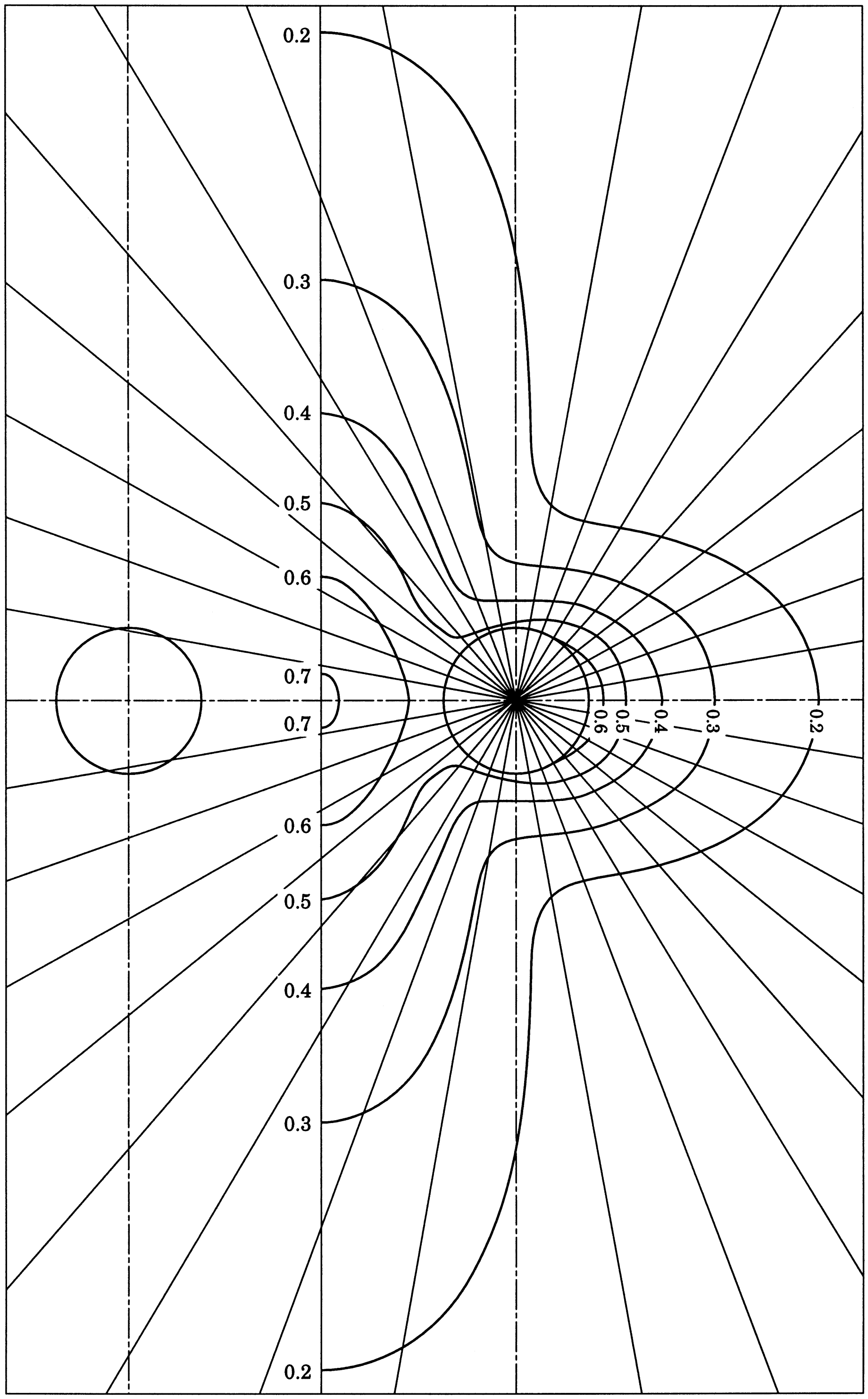

Figure 4. Equal pressure contours of two dilating spherical sources at the $z = 2a/3$ plane for $ka = 1$.

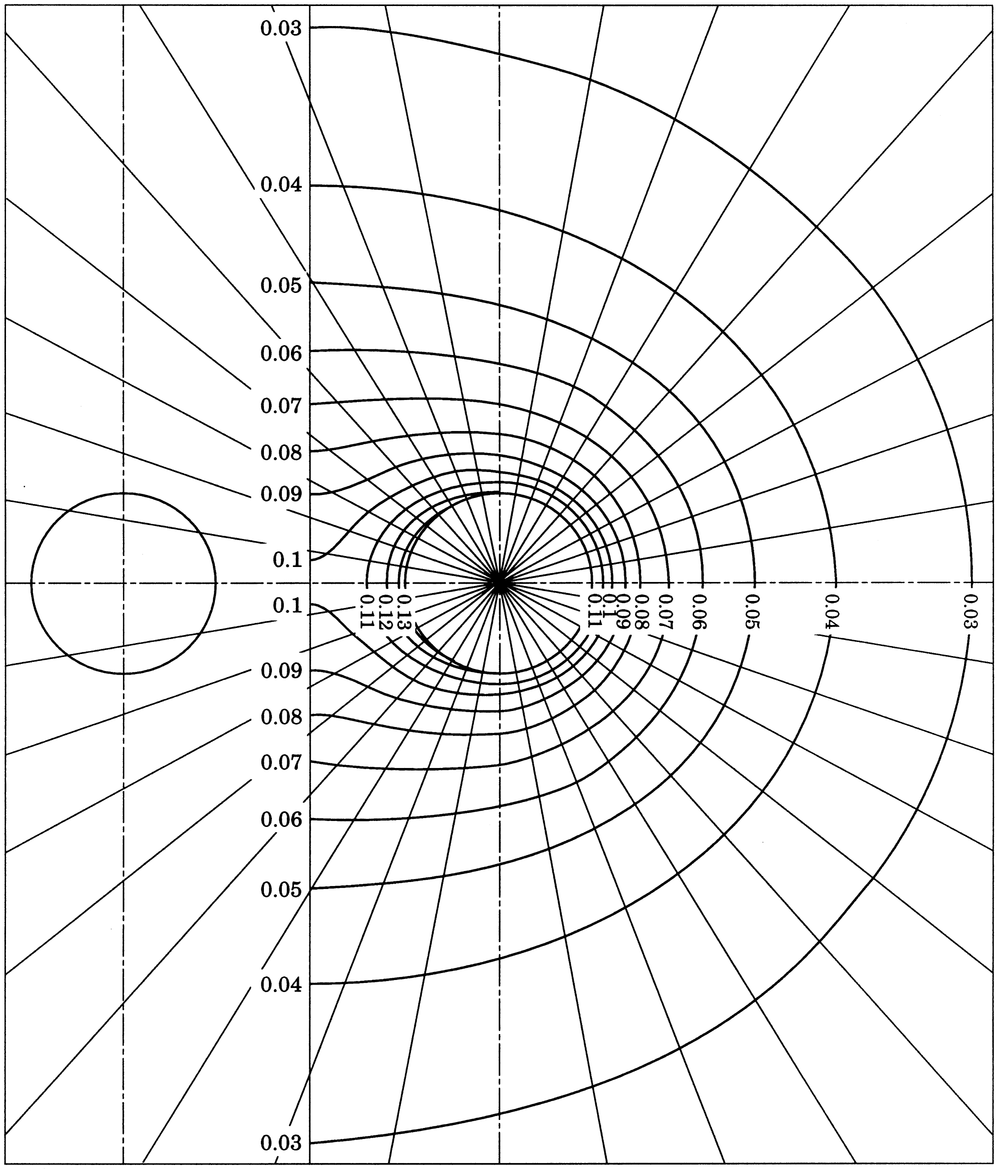

Figure 5. Equal pressure contours of two dilating spherical sources at the $z=0$ plane for $ka=0{\cdot}1$.

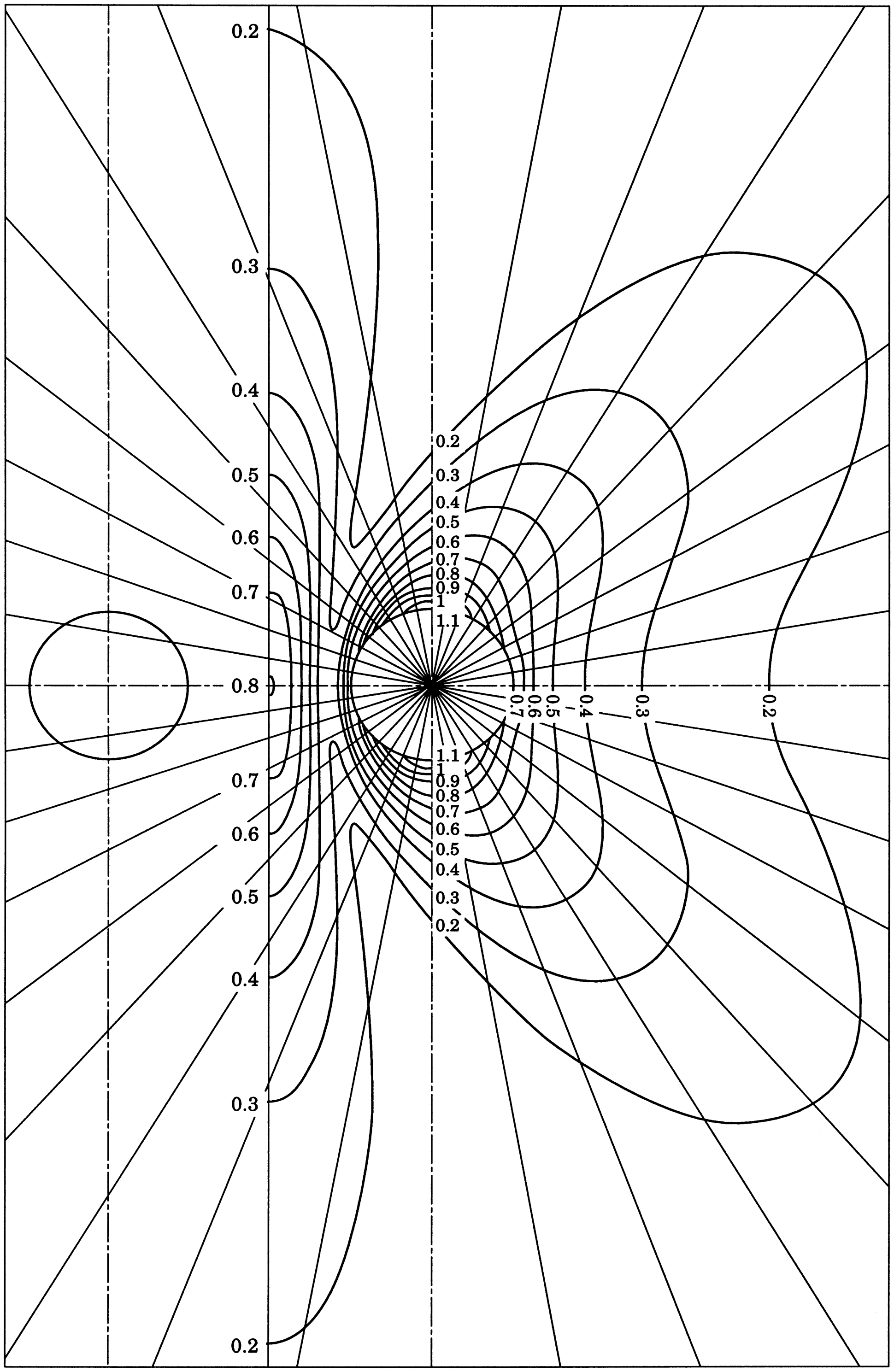

Figure 6. Equal pressure contours of two dilating spherical sources at the $z = 0$ plane for $ka = 2$.

the total number of nodes. Once the surface pressure distributions have been determined by equation (3), then computation of field pressures becomes possible by using the exterior Helmholtz integral: that is, equation (1) for **x** in the exterior. Previous applications for sound radiation from a single sphere show that equation (3) gives good results when using quadratic isoparametric elements [9, 11–13] and four-point Gaussian quadrature [12, 13].

The computer code which was developed during the course of this study has the capability to generate equation (3), for any number of vibrating three dimensional bodies of arbitrary shape, using quadrilateral quadratic isoparametric boundary elements and Gaussian quadrature with up to 256 integration points. The code was written in APL language and executed on an IBM-3090 computer. The precision of the code was validated using the known analytical results for the single dilating sphere and also the superposition solutions for the case of two spheres [14].

3. NEAR FIELD CHARACTERISTICS OF TWO DILATING SPHERES

Consider the problem of sound radiation from two spheres each of radius a and with centres at distance of $4a$, vibrating in phase radially with uniform normal surface velocity

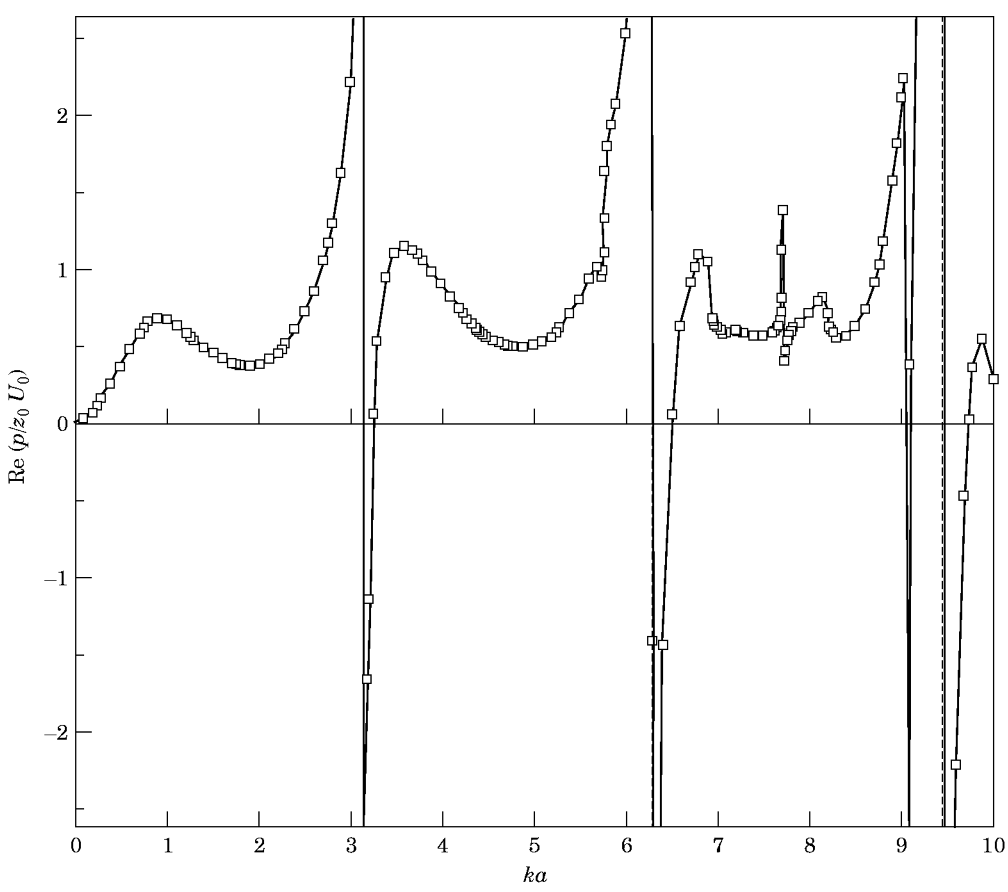

Figure 7. Variation of the real part of the non-dimensional surface pressure at the nearest points of the equatorial circles of two dilating spherical sources, computed by using SHIE, with the non-dimensional wavenumber ka.

amplitude U_0 and frequency f, as shown in Figure 1. Here, this problem is solved by using a 24-element boundary element model for each sphere, as shown in Figure A1 in the Appendix. Also presented in the Appendix are the local spherical co-ordinates of the 82 nodes for each sphere. Numerical integration was carried out by using 16-point Gaussian quadrature for every element. It should be pointed out that, since the spheres are symmetrical with respect to the x-z plane, the normal velocities of the particles in this plane vanish and the x-z plane behaves as a rigid plane satisfying the boundary condition $u=0$. Therefore, the solution of the two spherical dilating source problem includes the solution of the problem of a single dilating sphere in a 3-D half space given in reference [15].

Figures 2–6 show the equal pressure contours in several horizontal planes for various values of the non-dimensional wavenumber, ka. The numbers on the contours indicate the corresponding values of the non-dimensional pressure amplitude p/z_0U_0.

Figure 2 shows the contours in the vicinity of the equatorial circle (in the $z=0$ plane) of one sphere for a non-dimensional wavenumber $ka=1$. Since the sources are symmetrical with respect to the x-z plane, the equal pressure contours will also be symmetrical with respect to the x axis. Figures 3 and 4 give the contours in the vicinity of the circle of a sphere at the $z=a/3$ and $z=2a/3$ planes, for $ka=1$, respectively. It can be seen that acoustic pressures decrease as one moves up from the equatorial plane.

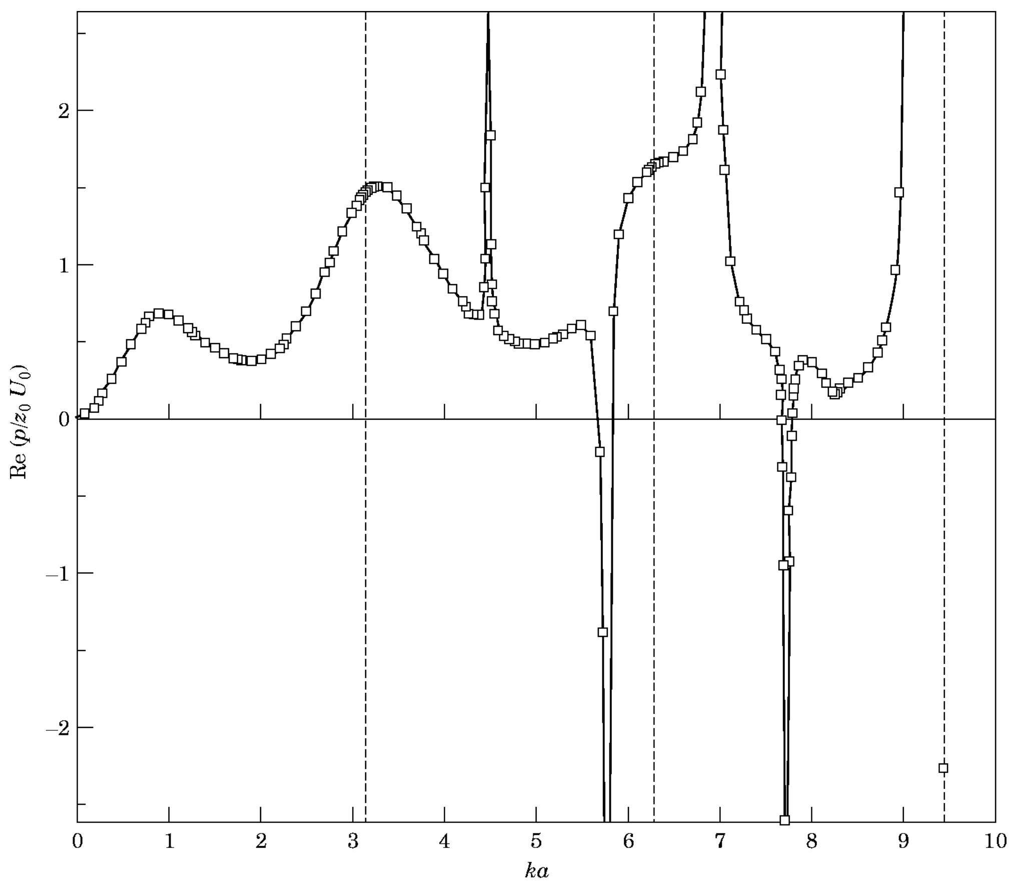

Figure 8. Variation of the real part of the non-dimensional surface pressure at the nearest points of the equatorial circles of two dilating spherical sources, computed by using CHIEF, with the non-dimensional wavenumber ka.

Figures 5 and 6 show the equal pressure contours at the $z = 0$ plane, for $ka = 0{\cdot}1$ and $ka = 2$, respectively. The influence of the frequency on the near field pressures can be inferred by comparing Figures 2, 5 and 6.

4. ACCURACY OF THE SOLUTIONS NEAR THE CRITICAL FREQUENCIES

The non-dimensional surface pressures of the dilating spheres were examined in a broad non-dimensional wavenumber spectrum ranging from $ka = 0{\cdot}1$ to $ka = 10$. The numerical results were computed by using the surface Helmholtz integral equation (SHIE) and also the combined Helmholtz integral equation formulation (CHIEF) [7]. The CHIEF method consists of the over-determination of the system of equations resulting from the surface Helmholtz integral with additional equations derived from the exterior Helmholtz integral for the interior. CHIEF yields good results in the neighbourhood of the critical frequencies and has received widespread use for sound radiation from single sources. In the case of a single dilating sphere, it is customary to use a single CHIEF interior point at the centre of the sphere [7, 9, 12]. Therefore, with two dilating spheres, CHIEF was first applied by taking two interior points, one at the centre of each sphere.

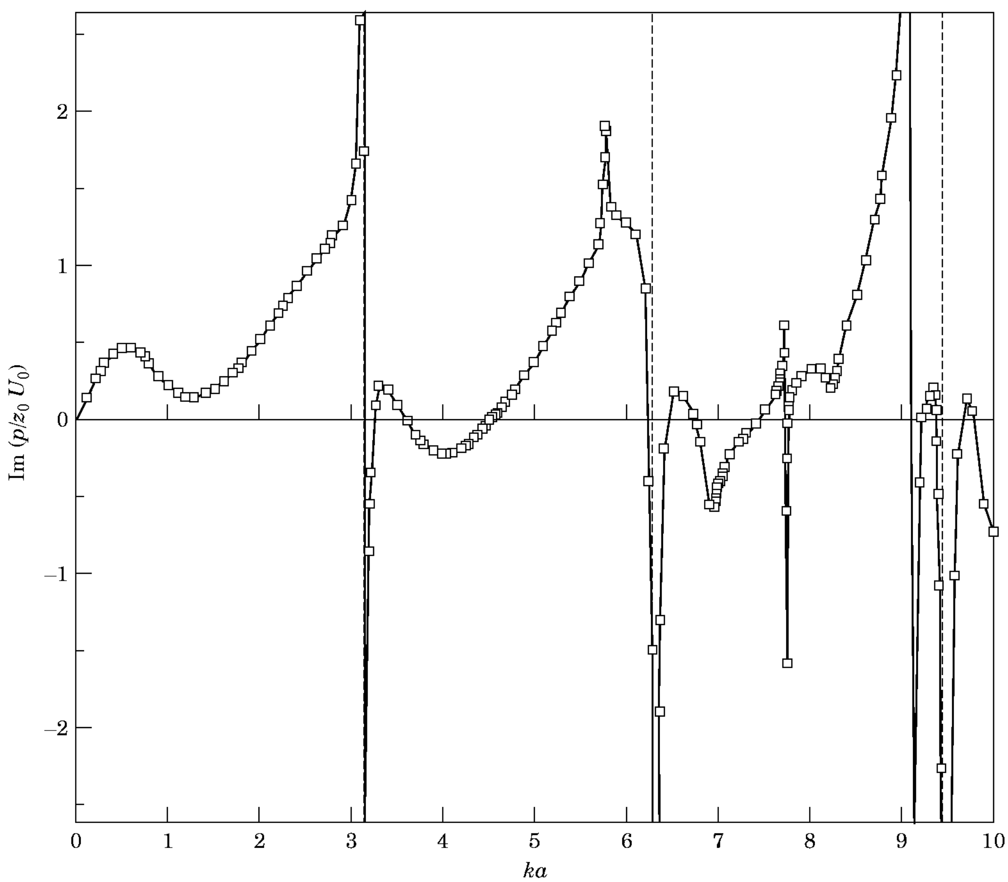

Figure 9. Variation of the imaginary part of the non-dimensional surface pressure at the nearest points of the equatorial circles of two dilating spherical sources, computed by using SHIE, with the non-dimensional wavenumber ka.

Figures 7 and 8 show the real parts of the non-dimensional surface pressure at the nearest points of the equatorial circles computed by using SHIE and CHIEF, respectively, as a function of the non-dimensional wavenumber ka. Figures 9 and 10 present the variation of the imaginary parts of the same pressures with ka. Figures 11 and 12 show the real parts of the non-dimensional surface pressure at the furthest points of the equatorial circles as computed by using SHIE and CHIEF, respectively. The imaginary parts of the same pressures are shown in Figures 13 and 14.

As can be seen from these figures, both SHIE and CHIEF give accurate results up to about $ka = 2{\cdot}5$ but for greater wavenumbers, the problem of non-uniqueness begins to arise. As is well established in the case of a single radiating body with a vibrating surface, these critical wavenumbers in the vicinity of which the non-uniqueness manifests itself are given by the characteristic wavenumbers of the interior problem for the same boundary with the Dirichlet boundary condition [7]. Hence, for a single sphere, the critical wavenumbers are $ka = \pi, 2\pi, 3\pi, \ldots$ for the spherically symmetrical modes and, in increasing order, $ka = 4{\cdot}493, 5{\cdot}763, 6{\cdot}988, \ldots$ for the non-spherically symmetrical modes of the associated interior problem. In all previous work concerned with the numerical implementation of the integral equation formulation to the radiation from a single dilating sphere, the

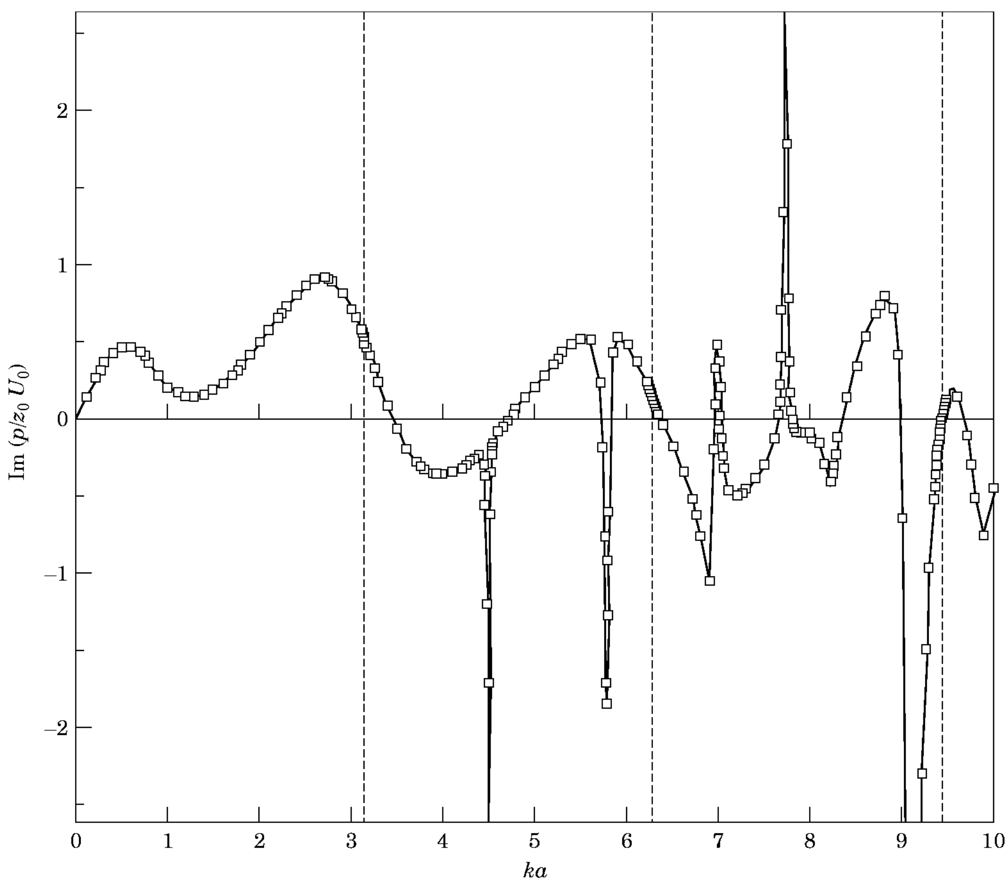

Figure 10. Variation of the imaginary part of the non-dimensional surface pressure at the nearest points of the equatorial circles of two dilating spherical sources, computed by using CHIEF, with the non-dimensional wavenumber ka.

non-uniqueness problem has been reported to arise only in the vicinity of the spherically symmetrical characteristic wavenumbers, that is, π, 2π, 3π, ... A mathematical proof of this has been provided by Copley [16].

In the case of sound radiation from vibrating multi-bodies, the critical wavenumbers will be given by the union of the characteristic wavenumbers of each body [17]. The two dilating sphere problem considered in this paper, is not spherically symmetrical and, even though the spheres are taken to be identical, the non-uniqueness problem is thus expected to occur around both the symmetrical and non-symmetrical characteristic wavenumbers of a single sphere, which are given above. Indeed, it is seen in Figures 7, 9, 11 and 13 that the non-uniqueness with SHIE occurs for the spherically symmetrical and non-symmetrical characteristic wavenumbers. It is, however, interesting to note that, for the smallest non-spherically symmetrical characteristic wavenumber, that is, $ka = 4{\cdot}493$, the non-uniqueness problem does not arise.

The CHIEF method, when applied by using a single interior point at the centre of each sphere, corrects the non-uniqueness at the spherically symmetrical characteristic wavenumber $ka = \pi$. It does not have any corrective effect at the non-spherically symmetrical characteristic wavenumber $ka = 5{\cdot}763$ and, furthermore, brings out the non-uniqueness at $ka = 4{\cdot}493$ to which SHIE is insensitive.

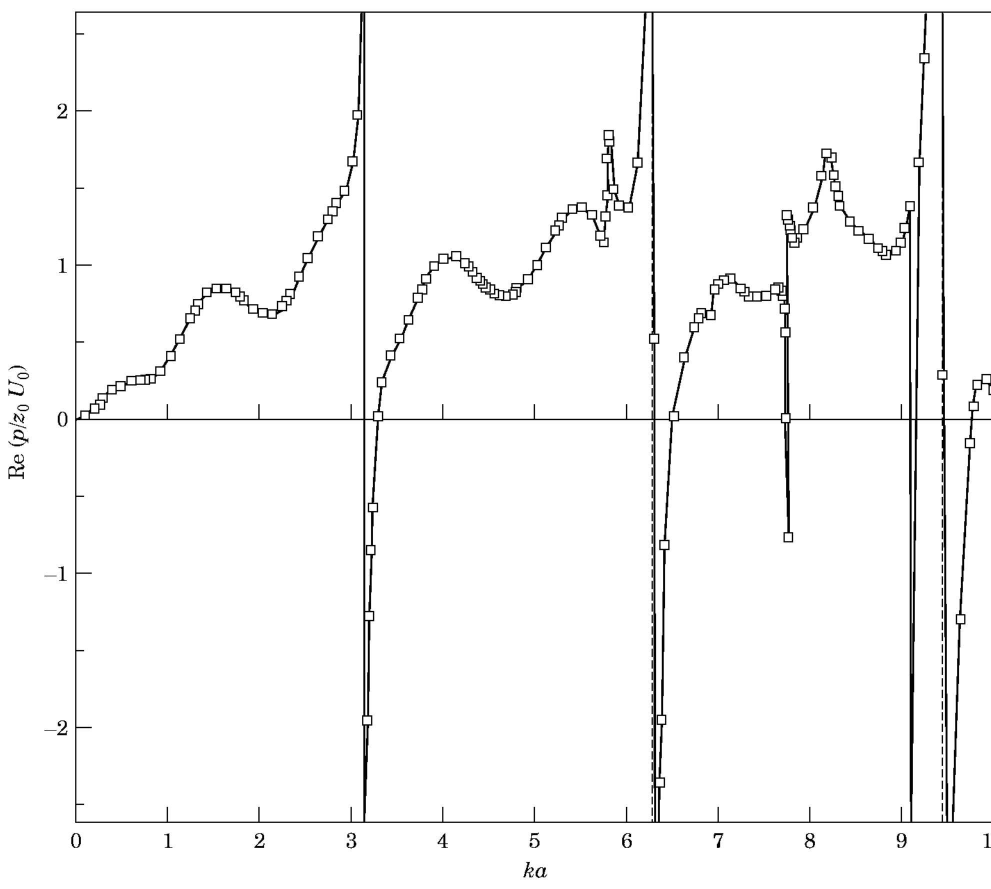

Figure 11. Variation of the real part of the non-dimensional surface pressure at the furthest points of the equatorial circles of two dilating spherical sources, computed by using SHIE, with the non-dimensional wavenumber ka.

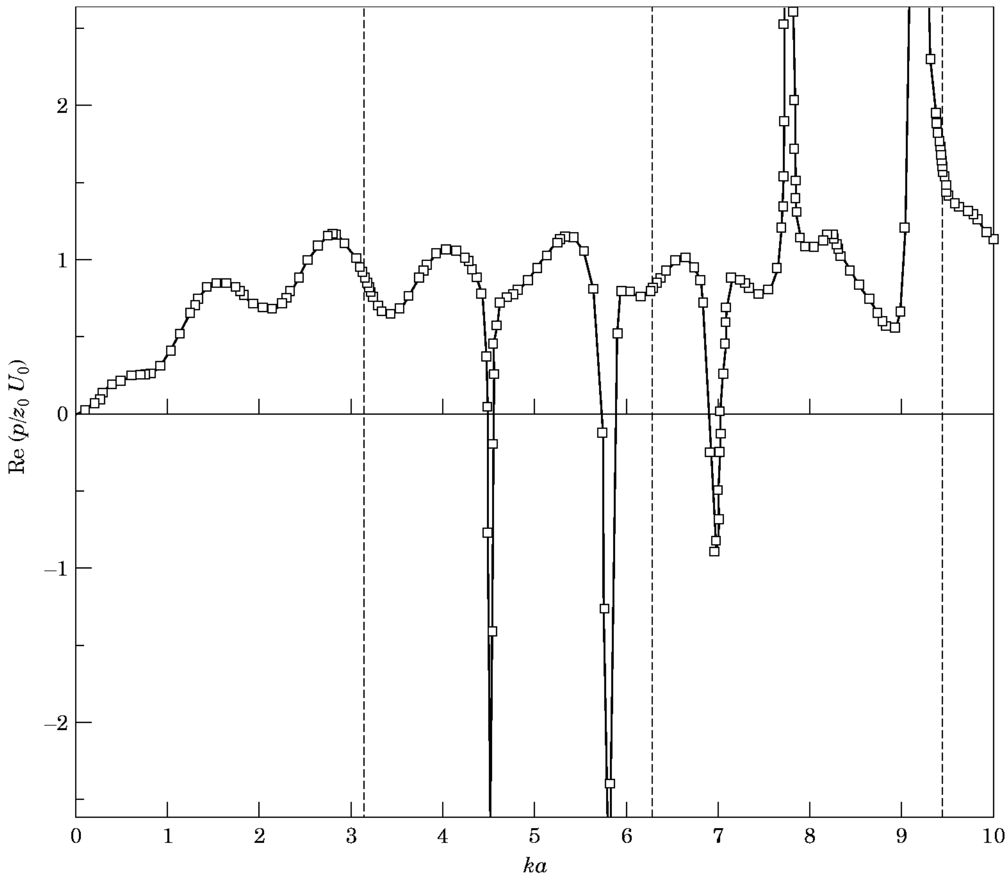

Figure 12. Variation of the real part of the non-dimensional surface pressure at the furthest points of the equatorial circles of two dilating spherical sources, computed by using CHIEF, with the non-dimensional wavenumber ka.

As is well known [12], CHIEF's performance depends on the location and number of the interior points, but the selection of the best points is basically a trial-and-error process. An attempt was made to improve CHIEF's performance by changing the positions of the interior points along the z axis. The four sets of points used were: (0, 0, 0·2; 0, 0, −0·2), (0, 0, 0·4; 0, 0, −0·4), (0, 0, 0·6; 0, 0, −0·6) and (0, 0, −0·8; 0, 0, 0·8). None of these points circumvented the occurrence of non-uniqueness satisfactorily around $ka=4{\cdot}493$. After a large number of tries with other sets of interior points, it has been found that the non-uniqueness around $ka=4{\cdot}493$ could be improved by using the interior points at (0·6, 0·5, 0·4; 0·3, 0·2, −0·1).

5. CONCLUSIONS

The near field and surface pressure characteristics of two dilating spheres were examined. The equal pressure contours of the spheres were presented for different horizontal planes and non-dimensional wavenumbers.

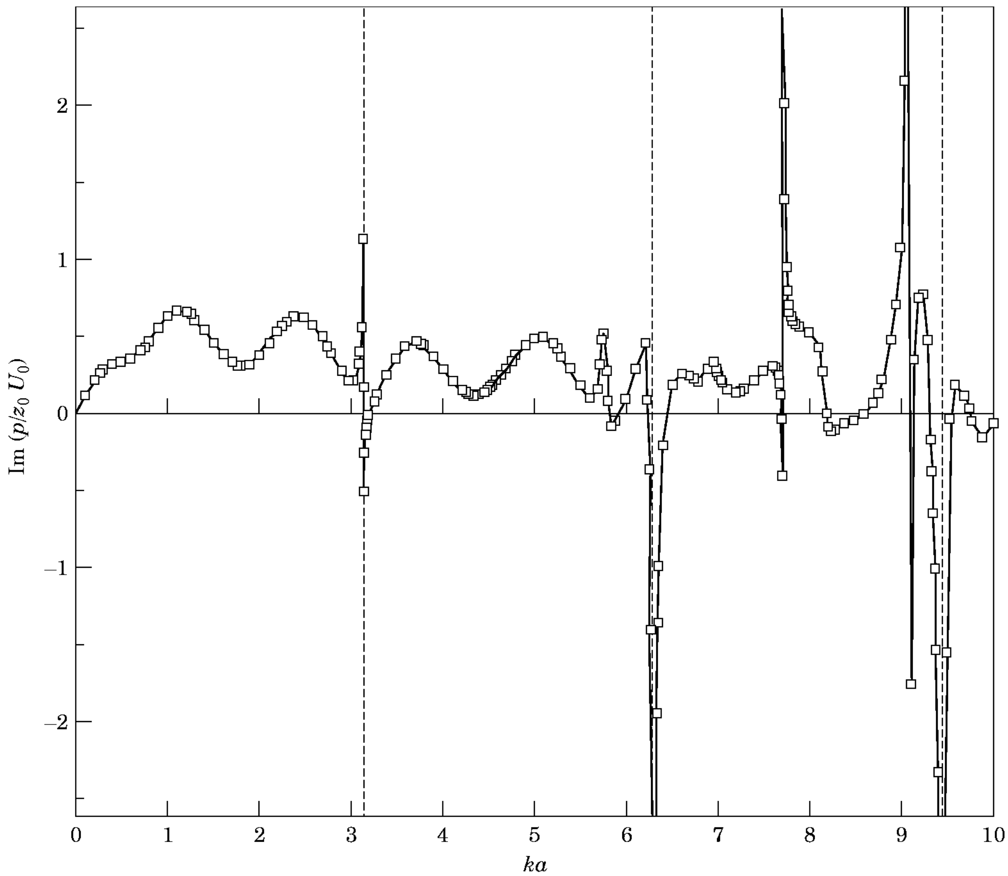

Figure 13. Variation of the imaginary part of the non-dimensional surface pressure at the furthest points of the equatorial circles of two dilating spherical sources, computed by using SHIE, with the non-dimensional wavenumber ka.

The surface pressure characteristics were obtained by using both the SHIE and CHIEF formulations. In respect to the effort required for circumventing the non-uniqueness at $ka = 4{\cdot}493$ by the CHIEF method, the best strategy for the problem under consideration would be to use CHIEF with interior points at the centre of the spheres to correct the non-uniqueness around $ka = \pi$ and to use SHIE for the range $\pi < ka < 2\pi$. For $ka > 2\pi$, neither SHIE nor CHIEF are reliable because of the increased density of the critical wavenumbers.

ACKNOWLEDGMENTS

The authors would like to acknowledge the comments by one of the referees which contributed to the discussion in section 4.

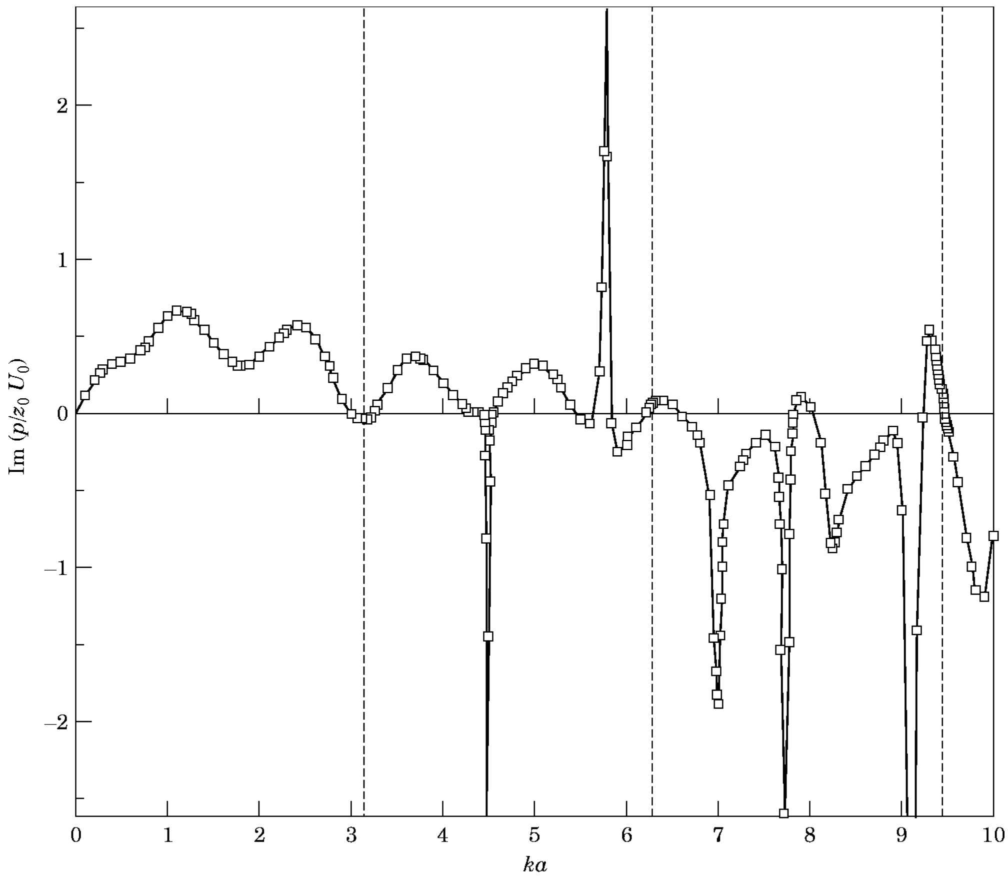

Figure 14. Variation of the imaginary part of the non-dimensional surface pressure at the furthest points of the equatorial circles of two dilating spherical sources, computed by using CHIEF, with the non-dimensional wavenumber ka.

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APPENDIX

The local Cartesian co-ordinates of the nodes (see Figure A2) are calculated in the computer code by the following co-ordinate transformation:

$$\bar{x}=r\sin\Theta\cos\Phi,\qquad \bar{y}=r\sin\Theta\sin\Phi,\qquad \bar{z}=r\cos\Theta. \tag{A1}$$

Appendix table overleaf

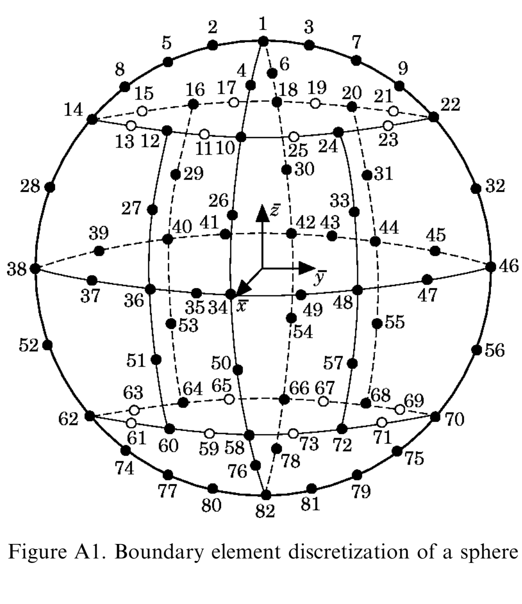

Figure A1. Boundary element discretization of a sphere

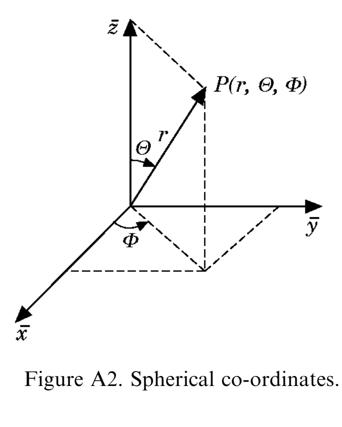

Figure A2. Spherical co-ordinates.

TABLE A1

Node	Local spherical co-ordinates of the nodes Θ^0	Φ^0
1	0	0
2	11·25	270
3	11·25	90
4	22·5	0
5	22·5	270
6	22·5	180
7	22·5	90
8	33·75	270
9	33·75	90
10	45	0
11	45	337·5
12	45	315
13	45	292·5
14	45	270
15	45	247·5
16	45	225
17	45	202·5
18	45	180
19	45	157·5
20	45	135
21	45	112·5
22	45	90
23	45	67·5
24	45	45
25	45	22·5
26	67·5	0
27	67·5	315
28	67·5	270
29	67·5	225
30	67·5	180
31	67·5	135
32	67·5	90
33	67·5	45
34	90	0
35	90	337·5
36	90	315
37	90	292·5
38	90	270
39	90	247·5
40	90	225
41	90	202·5
42	90	180
43	90	157·5
44	90	135
45	90	112·5
46	90	90
47	90	67·5
48	90	45
49	90	22·5
50	112·5	0
51	112·5	315
52	112·5	270
53	112·5	225
54	112·5	180

TABLE A1—*Continued*.

Node	Local spherical co-ordinates of the nodes	
	Θ^0	Φ^0
55	112·5	135
56	112·5	90
57	112·5	45
58	135	0
59	135	337·5
60	135	315
61	135	292·5
62	135	270
63	135	247·5
64	135	225
65	135	202·5
66	135	180
67	135	157·5
68	135	135
69	135	112·5
70	135	90
71	135	67·5
72	135	45
73	135	22·5
74	146·25	270
75	146·25	90
76	157·5	0
77	157·5	270
78	157·5	180
79	157·5	90
80	168·75	270
81	168·75	90
82	180	0