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BOOK REVIEWS

This issue's featured review by K. K. Tung is about SIAM's *Mathematics & Climate* by Kaper and Engler. This undergrad textbook should inspire students to think about the policy choices that could be made, based on sound math and statistical modeling and more specific conceptual models. Also included are reviews of E's *Principles of Multiscale Modeling*, Han and Wu's *Artificial Boundary Method*, Narang-Siddarth and Valasek's *Nonlinear Time Scale Systems*, Noonburg's *Ordinary Differential Equations*, Paul and Baschnagel's *Stochastic Processes*, Schuss's *Brownian Dynamics*, Shafarevich's two-volume *Basic Algebraic Geometry*, Shiryaev's *Problems in Probability*, Shtern's *Counterflows*, Strang's *Differential Equations and Linear Algebra*, and Syropoulos's *Theory of Fuzzy Computation*. You can count on quite a variety of opinions by our expert reviewers.

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Book Reviews

Edited by Robert E. O'Malley, Jr.

Featured Review: Mathematics & Climate. *By Hans Kaper and Hans Engler.* SIAM, Philadelphia, 2013. $59.00. xx+295 pp., softcover. ISBN 978-1-611972-60-3.

The science of climate change is complex and the field of research is expanding, increasingly drawing in mathematicians and statisticians. Yet the curriculum at the undergraduate level tends to lack material that "introduces students to mathematically interesting topics from climate science" and makes "climate issues understandable to readers coming from fields other than geophysics." This timely contribution from Kaper and Engler fills this gap and provides a textbook for undergraduates in mathematics and statistics wishing to explore climate sciences as an application area.

The writing style is concise and to the point. Complex subjects, such as the role of oceans in climate, atmospheric structure and circulation, and the cryosphere, are dealt with *descriptively*, with very brief but authoritative prose.

Some may say that the book is too ambitious: by dealing with so many complex topics in climate science, from the Earth's energy budget, the ocean conveyor belt, sea-ice shape, the transport of carbon dioxide, and plankton and algae, to El Niño, data inhomogeneity, and extreme events, the authors can only present these topics in a cursory manner. I think, however, that this is intended by the authors. The book is only meant to *expose* students to various interesting topics of current interest in climate science to which mathematical and statistical tools can be profitably applied. By showing how a few conceptual models utilizing only simple mathematics can be used to make sense of complex phenomena, the authors demonstrate the important role that mathematics and mathematicians can play in climate science.

The book's presentation of mathematical theory and techniques is interspersed among the climate topics, sometimes in separate chapters. Some mathematical topics are treated in detail, while others are dealt with briefly and descriptively. Dynamical systems theory has its own chapter and is discussed in detail, in the typical mathematician's definition-and-theorem format. This is followed by a chapter on bifurcation theory for equilibrium solutions of nonlinear differential equations in one and two dimensions. The subsequent two chapters deal with applications: a short chapter on the equilibrium solution of Stommel's two-box model of thermohaline circulation illustrates the possibility of multiple equilibria, bifurcation, and hysteresis. The following short chapter on the three-component Lorenz equations gives an example of an interesting dynamical system, using a numerical solution to show the strange attractors; however, few of the theorems from the dynamical systems chapter are used.

My favorite chapter is Chapter 11, "Fourier Transforms." The chapter presents Fourier series as a trigonometric interpolation and then proceeds to derive the dis-

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crete Fourier transform, leading to a discussion of the advantages of the fast Fourier transform (FFT). The FFT is then used to analyze some sample time series, showing that a different perspective can be obtained by examining their power spectra. Correlation and autocorrelation are defined and interpreted. An excellent section on Milankovitch's theory of glacial cycles follows as an application, including a spectral analysis of the forcing function and the observed response.

The chapters on statistical techniques are useful, as they are commonly used to analyze climate time series of observation and model data. Such datasets are nowadays freely available online, and so a student can actually apply the techniques learned to real problems. Chapter 9 very briefly presents regression analysis, which is then applied in Chapter 10 to the carbon dioxide data from Mauna Loa. Chapter 19 presents theorems from statistics that can be used to infer whether or not an observed incidence of extreme events is random. Chapter 20 discusses various data assimilation methods, including the Bayesian approach. The presentation of the statistical results is very brief, which might be due to the perception of the authors that students taking this course probably have already acquired these concepts from other statistics courses.

The long Chapter 14 derives the hydrodynamics equations (partial differential equations) governing fluid flows on a rotating sphere. These are the equations used in general circulation models, which are supposedly the subject of the following chapter on climate models but are not used there. Instead, the authors present arguments for viewing such models as a dynamical system in functional spaces, resulting in an "abstract climate model." No further insight is gained from this viewpoint, other than the fact that when spectrally truncated, the Rayleigh–Bénard convection equations can lead to the three-component Lorenz equations, which form a dynamical system.

Chapter 16, on the El Niño Southern Oscillation (ENSO), returns to the "conceptual models" used so well in the rest of the book to explain the mechanisms behind this quasi-periodic climate pattern that occurs across the equatorial Pacific Ocean. A conceptual model is not derived from the governing partial differential equations, or from first principles, but is instead argued for as being reasonable and plausible. A recharge-oscillator model of Jin and a delayed-oscillator model of Battisti and Hirst are presented, and the former is solved numerically. For the latter, further discussion of Rossby wave and Kelvin wave dispersions is given but does not emphasize their ties to the life cycle of ENSO and its observed period.

Chapter 12, "Zonal Energy Budget," does a thorough job of deriving the energy balance model of the longitudinally averaged Earth. In contrast to the brevity of earlier and later chapters, each topic here is explained in detail, even including a section on the Legendre polynomial expansion of the solutions. As was the case for Chapter 14, this long derivation is also not made use of later. After completing the model by fitting the model parameters to the present climate, the chapter ends. Subsequent chapters never return to the equation derived.

In summary, *Mathematics & Climate* is a delightful short book at the intersection of mathematics and climate science. It serves its purpose well as an excellent textbook for a one-semester course, especially if the instructor has disciplinary knowledge of the topics in climate science.

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Problems in Probability. *By Albert N. Shiryaev.* Springer, New York, 2012. $79.95. xii+427 pp., hardcover. ISBN 978-1-4614-3687-4.

The author A. N. Shiryaev is a distinguished mathematician of the famous Russian school of probability. The present book is a rich and comprehensive collection of problems compiled over many years for use in graduate courses at Moscow State University and other academic institutions in Russia. These courses are generally based on books like the author's two-volume *Probability* [6], referred to as $\underline{P}$, whose earlier editions have been translated into English [5]. However, *Problems in Probability* can be used to aid the instruction of a variety of courses in probability in U.S. universities and elsewhere. Although there are frequent references to the books $\underline{P}$, one can for the most part easily figure out the underlying contexts, notation, and definitions without looking up these source books. The first chapter, "Elementary Probability Theory" reminds one of Feller's classic [3]. It provides a host of combinatorial identities, some basic and some special, used for solving interesting problems in finite probability. In particular, they lead to the derivation of the reflection principle for the simple symmetric random walk, the arcsine law, and the convergence of the binomial to the Gaussian and the Poisson. Finite (and countable) state Markov chains also appear briefly here and more elaborately in the last chapter (Chapter 8), "Sequences of Random Variables that Form Markov Chains." This material, together with the weak convergence theory and its application to the convergence of random walks to the Brownian motion appearing in Chapter 3, could be used to supplement a first graduate course on stochastic processes. Such a course is, however, generally preceded by a basic graduate course in measure theoretic probability following a text such as [1] or [2]. The instructor of such a course will find Chapters 2–5 and 7 of this book to be of great use. Finally, the brief Chapter 6 on the L^2 theory of wide sense stationary processes contains a number of important exercises, including several on the Kalman filter.

It should be mentioned that many of the exercises in the present book introduce important topics in probability in a self-contained manner, but the majority of the problems test the student's understanding of basic concepts. Some of the problems are relatively simple, while others are quite challenging. Here are some examples, focusing primarily on the basic Chapter 2 ("Mathematical Foundations of Probability Theory," pp. 59–179).

(1) Problems 1.2.28–29 (referring to exercises 28, 29 from Chapter 1, section 2 in $\underline{P}$) are on Polya's urn scheme.

(2) Problem 2.1.26 asks for an example of two finite measures μ_1, μ_2, where the smallest measure ν satisfying $\nu \geq \mu_1$, $\nu \geq \mu_2$ is $\mu_1 + \mu_2$ and not $\max(\mu_1, \mu_2)$. This problem should perhaps have been placed a few sections later after the introduction of the Radon–Nikodým theorem, where the following simple result could be obtained: If f_1 and f_2 are the densities of μ_1 and μ_2 (with respect to some sigma-finite measure μ, say, $\mu = \mu_1 + \mu_2$), then ν is the measure whose density is $\max(f_1, f_2)$. Hence $\nu = \mu_1 + \mu_2$ only if μ_1 and μ_2 are supported on disjoint sets.

(3) Problems 2.6.84–85 introduce the so-called ladder epochs $T_1, T_2, \ldots$ of a (general) random walk $S_n = \xi_1 + \xi_2 + \cdots + \xi_n$ ($S_0 = 0$). That is, $T_0 = 0$, $T_k = \inf\{n > T_{k-1} : S_n - S_{T_{k-1}} > 0\}$, $k \geq 1$. One has to prove $P(T_1 < \infty) = 1$ if $B \equiv \sum_{1 \leq n < \infty} P(S_n > 0)/n = \infty$, and $P(T_1 < \infty) = 1 - \exp\{-B\}$ if $B < \infty$. This is an important result, essentially due to Spitzer, with many implications (see Feller [4, XII.7]).

(4) Problem 2.10.57. Let $\xi_1, \xi_2, \ldots$ be a positive i.i.d. sequence with a (common) density f satisfying $\lim_{x \downarrow 0} f(x) = \lambda > 0$. Then $n[\min(\xi_1, \xi_2, \ldots, \xi_n)]$ converges in distribution, as $n \to \infty$, to an exponentially distributed random variable with parameter λ.

(5) Problem 2.10.61. Show that the Lebesgue measure of the points in $[0, 1]$, such that the nth term in its continued fraction expansion equals k, converges to $(1/\ln 2)\ln[\{1 + 1/k\}/\{1 + 1/(k + 1)\}]$ as $n \to \infty$, $k = 1, 2, \ldots$. This again is an important basic result in the theory of con-

tinued fractions, which probably belongs more appropriately to Chapter 5 ("Stationary (in Strict Sense) Random Sequences and Ergodic Theory").

(6) Problem 4.5.9. Let $M_n = \max(\xi_1, \xi_2, \ldots, \xi_n)$, where $\xi_1, \xi_2, \ldots$ are i.i.d. standard Cauchy. Show that $P(M_n/n \le x) \to \exp\{-1/\pi x\}$ as $n \to \infty$, for $x > 0$.

(7) Problem 7.12.8. Derive the Black–Scholes formula for European-style call options with terminal payoff $(S_T - K)_+$, where $S_t = S_0 \exp\{t\mu + \sigma W_t\}$ and $\{W_t\}$ is a standard Brownian motion.

There are some minor typos in the text, but none very serious.

Problems in Probability is an excellent source of exercises for graduate courses in probability. The exercises are diverse and very well chosen, and include both relatively simple ones testing the student's understanding of basic concepts and ability to apply them, as well as challenging ones which may be assigned as longer projects and on which the instructor may also hone his/her expertise.

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Artificial Boundary Method. *By Houde Han and Xiaonan Wu*. Springer, New York, 2013. \$129.00. viii+423 pp., hardcover. ISBN 978-3642354632.

The theme of this book concerns the following type of question. Consider a boundary value problem, or an initial boundary value problem, in an unbounded spatial domain D. Suppose we truncate the unbounded domain by introducing an artificial boundary B, which encloses a finite subdomain Ω. Can we find a boundary condition on B such that the original problem in D and the new problem in Ω are equivalent? By "equivalent" we mean that the solution of the problem in Ω is exactly the restriction (to Ω) of the solution of the problem in D. Such a boundary condition is called an exact *artificial boundary condition* (ABC). The authors omit the word "exact" throughout the book, but when referring to ABCs they always mean *exact* ABCs; otherwise, they use the phrase "approximate ABCs." Other names are also used in various fields of applications; for example, in geophysics it is customary to call such a condition an exact absorbing boundary condition (the acronym ABC still applies). In almost all cases where such an exact ABC can be found (except for the 1D linear wave equation, which is especially simple) this is a global (i.e., nonlocal) boundary condition. A typical form of an exact ABC for a boundary value problem is

$$\frac{\partial u}{\partial n}(\boldsymbol{x}) = \int_B m(\boldsymbol{x}, \boldsymbol{x}')u(\boldsymbol{x}')\, d\boldsymbol{x}', \quad \boldsymbol{x} \in B\ ,$$

which is called the Dirichlet-to-Neumann (DtN) boundary condition because it maps the Dirichlet datum u to the Neumann datum $\partial u/\partial n$ on B. The DtN operator is also called the Steklov–Poincaré operator, which is the name used in this book.

The authors of this book, Han and Wu, and also their coworkers Bao, Yu, Zheng, and others, are among the few groups of researchers who have done a lot of excellent work, mostly theoretical, on this subject. This book is a summary of their work; it is an important contribution, and its assembly in book form is beneficial to those who would like to study the subject. In particular, the book offers several useful mathematical tools for developing and analyzing exact ABCs, and their semianalytical and approximate versions, for various types of PDEs.

It should be noted that although the book was published in 2013, it is not up-to-date, in that it does not present the state of the art of ABCs in general. The field saw a true revolution in the mid-1990s, with the development of high-order (local) ABCs [1] and the perfectly matched layer (PML) [2]. A large volume of literature exists dealing with extensions, analysis, improvement, and application of high-order ABCs and PMLs. Moreover, the use of these techniques in scientific and industrial applications today is far wider than that of exact nonlocal ABCs of the type studied in this book. In addition, the book mentions work from the 1970s on infinite elements (in Chapter 6) but ignores the much improved (and mathematically more correct) infinite elements developed by Burnett and by Astley's group since the 1990s. I would have expected the authors to at least mention these important developments of the last 20 years in the introduction, but the book does not cover them at all.

What I see as another slight weakness of this book is the fact that although the subject matter is a computational method, the book is totally theoretical and lacks any numerical examples or tests. In some parts of the book the authors discuss discretization methods for the problems studied, such as finite differences and finite elements, but with no numerical demonstration or verification. This is surprising, since the authors' papers *do* include numerical examples, which shows that they do not belong to the small group of researchers in scientific computing who believe that numerical verification is superfluous.

Chapter 1 discusses exact ABCs for second-order scalar elliptic equations: Poisson's equation, the modified Helmholtz equation and the Helmholtz equation, in two and three dimensions. Stability and error estimates are proved for the two former cases. I was a bit disappointed not to find error analysis for the Helmholtz equation, since this is the more mathematically challenging and interesting case due to the lack of coercivity. The following three chapters discuss exact ABCs for elastostatics and for the Stokes problem (Chapter 2), for the heat equation and for the linear Schrödinger equation (Chapter 3), for the wave equation, for the Klein–Gordon equation, and for the KdV equation (Chapter 4). Chapter 5 discusses the localization of the nonlocal conditions of the previous chapters, and error estimates are provided. Chapter 6 deals with "discrete ABCs," which are semianalytical ABCs designed after partial discretization. These are very useful for those cases in which an exact analytical ABC is not available, such as the case of the elastic half-space. Chapter 7 discusses "implicit ABCs," which are boundary integral equations on an artificial boundary. Chapter 8 treats ABCs for nonlinear problems, namely, the Burgers equation, the KPZ equation, and the nonlinear Schrödinger equation. The Cole–Hopf transform is used as the main mathematical tool. Chapter 9 is unique in that it does not treat an unbounded domain problem, but instead problems with a geometrical singularity like a reentrant corner and an interface joint. It is known that the same sort of techniques that are useful for unbounded domain problems can be applied to such geometrically singular problems. Whereas in the former case the unbounded domain is eliminated by the use of the ABC, in the latter it is the singularity region that is eliminated.

The style of writing is nice: it is rigorous without being dry. The text is clear and easy to read. The book's format is pleasant to the eye and inviting. The different chapters are quite independent, but are written in a uniform style. Each chapter ends with an alphabetical list of references for that chapter, which is convenient. However, there is no subject index at the end of the book. This is not a major deficiency owing to the very clear structure of the book's chapters.

In summary, this book is a useful assembly of the authors' work on exact ABCs and related topics, which provides the reader with a collection of mathematical tools for the design and analysis of such ABCs. If the authors ever publish a second edition, I would suggest adding a chapter to discuss modern developments like PMLs and high-order (local) ABCs, adding numerical examples to all the chapters, and adding an index.

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Theory of Fuzzy Computation. *By A. Syropoulos*. Springer, New York, 2014. $109.00. xii+162 pp., hardcover. ISBN 978-1-4614-8378-6.

One of the starting ideas for Zadeh's introduction of fuzzy sets was, besides the idea that these fuzzy sets should allow for simpler model building processes for complex systems, the hope that together with such simplifications new types of algorithms would allow for simpler calculations.

Heuristically this proved to be the case, but the mathematical understanding of the complexity of computations is strongly related to recursion theory. However, such a relationship of fuzzy sets to recursion theoretic considerations is far from trivial as the standard understanding of fuzzy sets uses the real unit interval as the structure of membership degrees, and recursion theory with reals is a notoriously complicated matter.

As for many fuzzified notions from classical mathematics, the first ideas for fuzzy algorithms and fuzzy Turing machines came into consideration around 1970, but the topic never reached the level of mainstream interest. Nevertheless, from time to time interesting new ideas arose to develop the field. Thus, it is not astonishing that the book under consideration is the first to be strongly devoted to this topic. The author intends to cover the basic approaches and results and thus offers an interesting survey of the field—a survey that has been missing up to now.

The book's formal considerations start, after a very general and philosophically oriented first chapter, in Chapter 2 with "A Précis of Classical Computability Theory." In a quite concise manner this chapter starts with Turing machines and Kolmogorov–Uspensky algorithms, continues with recursive functions and relations, and finishes with some remarks on computational complexity.

Similarly concise, the "Elements of Fuzzy Set Theory" are discussed in Chapter 3, covering the most basic notions of fuzzy sets and fuzzy relations up to a definition of t-norms and t-conorms.

Chapter 4, "On Fuzzy Turing Machines," and Chapter 5 on "Other Fuzzy Models of Computation" present the core material of the book. They are followed by two appendices on "Computing with Words" and on "The Rough Set Approach." The book closes with a well-constructed list of references and with a subject and a name index.

Chapter 4 starts from the work of E. E. Santos in the early 1970s, discusses its evolution, and follows that line of approach up to fuzzy Turing-W-machines. Considerations of the computational power of fuzzy Turing machines follow and lead to discussions about universal fuzzy Turing machines. The focus then moves to fuzzily recursive sets and to effective domains and fuzzy sets. Some final considerations on extensions to L-fuzzy sets and, very briefly, on fuzzy complexity theory close this chapter.

Chapter 5 discusses fuzzifications of some more nonstandard approaches to computations, partly inspired by natural phenomena. Four of them form the main focus: (i) a generalization of the idea of membrane computing via P-systems, (ii) fuzzy labeled transition systems, (iii) fuzzy X-machines, and (iv) a fuzzy version of the chemical abstract machine.

The book nicely explains the state of the art of its field. What is missing is a critical comparison of the different approaches and a clear evaluation of their respective benefits, disadvantages, and problems.

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Basic Algebraic Geometry. Volumes 1 and 2. Third Edition. *By Igor R. Shafarevich*. Springer, New York, 2013. Vol. 1: $79.99. xviii+310 pp., hardcover. ISBN 978-3-642-37955-0. Vol. 2: $69.99. xiv+262 pp., hardcover. ISBN 978-3-642-38009-9.

When this book first appeared (Russian 1972; English translation 1974), there was no other overall introduction to algebraic geometry. As the author said in his preface, "The aim of this book is to set forth the elements of algebraic geometry to a fairly wide extent, so as to give a general idea of this branch of mathematics, and to provide a basis for the study of the more specialized literature." His concept of "a fairly wide extent" led him to devote about half the book to varieties over a field, followed by a short section on schemes and then a selection of topics concerned with the topology of algebraic varieties over the complex numbers, and their relation to complex manifolds.

By the time of the second edition (1988), many other books had appeared introducing various aspects of algebraic geometry, but, as the author points out, none of them held the same aim of providing an overall view without going into too much detail. The second edition and the present third edition reinforce this intention, adding new material to broaden the scope and increasing the size of the book by about 40%. Still, the author manages to adhere to his original principle of explaining results from the beginning with a minimum of reliance on the machinery of commutative algebra, topology, sheaf theory, and so forth.

We owe a great deal to the translator, Miles Reid, himself a distinguished mathematician, for his careful translation into the language of current English-speaking algebraic geometers and for pointing out further references to the literature in English. In his "Translator's Note," he gives his own opinion of the book: it has "a well-considered choice of topics, with a human-oriented discussion of the motivation and the ideas, and some sample results (including a good number of hard theorems with complete proofs)." He goes on to say "the student who wants to get through the technical material of algebraic geometry quickly and at full strength should perhaps turn to Hartshorne's book [1]; however, my experience is that some graduate students ... can work hard for a year or two on Chapters 2–3 of Hartshorne, and still know more-or-less nothing at the end of it. ... For all such students, and for the many specialists in other branches of math who need a liberal education in algebraic geometry, Shafarevich's book is a must."

Let us now look at the content of the book in some more detail. The first part, "Varieties in Projective Space," is now volume one of two. The basic notions of a variety as defined by polynomial equations in an affine space, their rational functions, maps, projective and quasi-projective varieties, and the notion of dimension, can be found in many books. What distinguishes this treatment is the absence of any reliance on results of commutative algebra, except perhaps for Hilbert's Nullstellensatz. Also remarkable is the introduction at an early stage of some important results, such as the fact that a nonsingular plane cubic curve is not rational, or the Grassmann variety parametrizing lines in projective 3-space. This avoids the potential dryness of more formal treatments.

New material in this edition includes a full discussion of plane cubic curves, with the class group, the group law, and their characterization as those nonsingular curves for which the dimension of the linear system is one less than the degree for every effective divisor D on the curve.

Another major addition to this third edition is a complete proof of the Riemann–Roch theorem for curves. The proof given here is the old algebraic proof using distributions, improved by the use of Tate's theory of residues. The proof is "elementary" in the sense of not using any fancy machinery, but (in my opinion) mysterious in using strange definitions and constructions that appear nowhere else. It is one of those proofs where you can follow every step, but at the end have no understanding of why the result is true. I find this disappointing, since the proof using cohomology and Serre duality (see, for example, [1, Chap. IV, section 1]) is so beautiful and so simple. However, the cohomology theory of coherent sheaves falls outside the scope of this book.

Volume 2, "Schemes and Complex Manifolds," corresponds to Parts 2 and 3 of the first edition. Chapter 5 gives the definition and basic properties of schemes, and Chapter 6 uses these notions to study abstract varieties, defined as reduced separated schemes of finite type over an algebraically closed field k. This treatment of schemes is only a beginning, avoiding, for example, a general discussion of separated and proper morphisms. A significant inclusion here is a discussion of flat families, the Hilbert polynomial, and the Hilbert scheme parametrizing closed subschemes of projective space.

The last chapter of Volume 2 treats the topology of algebraic varieties over the complex numbers and some topics on complex manifolds. For these chapters, the author does assume some knowledge of differentiable manifolds, analytic functions, and ordinary homology and cohomology. For the newcomer to abstract algebraic geometry, this discussions based on more familiar complex manifolds will surely be helpful. New in this third edition is also a section on Kähler manifolds.

Overall, I find the book wonderfully put together, and I am sure the reader will learn a lot, either from systematic study or from browsing particular topics. There are a few glitches, as one might expect in such a large book. For example, the work of Tate and Arbarello et al. mentioned in the section on the Riemann–Roch theorem seems to have escaped inclusion in the bibliography. However, what bothered me most is the numbering of results, which is awful. In each chapter, the theorems, propositions, corollaries, examples, remarks, etc., each have their own independent numbering system, running consecutively throughout the chapter. This makes it a real chore to track any internal reference in the book.

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Nonlinear Filtering and Optimal Phase Tracking. *By Zeev Schuss.* Springer, New York, 2012. $74.95. xviii+262 pp., hardcover. ISBN 978-1-4614-0486-6.

Papoulis's books on probability and control theory have occupied the bookshelves of electrical engineers, applied mathematicians, physicists, and students of many other disciplines for several decades now. They offer a direct presentation of probability, statistics, and applications to signal processing. So what can we expect that is new from Zeev Schuss's recent book *Nonlinear Filtering and Optimal Phase Tracking*?

Suppose that in order to sell your cellular communication start-up company for tens of millions of U.S. dollars, you need to convince your prospective investors that the new and revolutionary signal tracking method that you have designed outperforms all existing methods and works at low SNRs hitherto unheard of. For example, its mean time between failures (MTBF) is longer than a year. Obviously, you cannot afford to construct a physical demo system and run it for over a year, while the investors and their hundreds of millions dollar sit idle and wait. Equally, no computer can run a simulation of a cellular telephone company's traffic for a year. Here, mathematical modeling, analysis, and simulation theory come to the rescue. You can show the investors that optimizing the MTBF (Chapter 7) rather than the expected mean square estimation error produces an estimator that outperforms the classical phase lock loop (PLL) with respect to the MTBF by many orders of magnitude. For example, it doesn't lose lock at SNRs that are 8–12dB lower than in the PLL. You can convince the investor by running a short simulation at low SNR that does not require a year to encounter loss of lock. Once the simulation shows the validity of the high SNR analysis also at low SNR, even nonbelievers in mathematical proofs should see the light.

The key mathematical issue here is the asymptotic estimate of the mean time to loss of lock (MTLL) (section 7.6), which is an estimate of the mean first passage time (MFPT) of noisy dynamics from an attractor of the noiseless dynamics. The latter blows up exponentially as the noise amplitude tends to zero. The novelty and the strength of the book lie in the development of a new mathematical method for the derivation of Zakai's stochastic partial differential equation (PDE) for the a posteriori probability density function of the estimation error in its Stratonovich form (Chapters 3 and 7) and of an analytical method for its asymptotic solution at low noise.

The book explains how to derive the Fokker–Planck equation using path-integrals (the classical material in the first three chapters is based on Brownian motion and Itô's calculus). At this stage, it is thus possible to express the probability of not losing lock, conditioned on a priori observations. The book explains how the conditional MFPT can be estimated using the WKB approximation (section 4.1), and how the WKB solution leads to a set of stochastic equations that represent realizable filters. Although the asymptotic theory is classically used in diffusion theory and quantum mechanics, the application of the new methods to communications theory resolved, among others, the malignant cubic sensor problem that previously withstood all solution efforts (section 4.2).

Another difficulty solved in the book is the asymptotic matching of the large deviations approximate solution (which is the WKB approximation) to boundary conditions. The book presents a large ensemble of new solutions to classical problems in signal processing and filtering theory. Graphs are presented in color. The most original and new methods are dispersed across different chapters.

Chapter 1 presents the classical elementary notions of Brownian motion and stochastic processes based on Itô's calculus, which links them to the PDE approach, e.g., the Andronov–Vitt–Pontryagin equation. Interestingly, Part 1.7 explains how to construct a Markov process with a prescribed spectrum based on continuous fraction representation of the Laplace transform of the autocorrelation function. The case of the $1/f$ noise is analyzed in great detail.

In the next chapter, the connection between discrete and continuous stochastic processes is made by analyzing Euler's scheme with a path-integral. This allows the derivation of boundary conditions and limit equations for probabilities and, in particular, to detect boundary layers that need to be resolved. This is a key necessary introduction to Chapter 3 on nonlinear filtering, which presents the classical stochastic equations for the minimal a posteriori mean square error estimator and for the a posteriori density or the energy functional associated with the problem.

Chapter 4 is dedicated to the solution of Zakai's equation using asymptotic analysis of PDEs. A particular case is the solution of the cubic sensor problem, which is a benchmark case for nonlinear filtering that was first posed by Bucy in 1969 and then taken up by many.

Chapter 5, the analysis of the MTLL in the two-dimensional PLL using the Hamilton–Jacobi equation (HJE), is very instructive. It shows how to use numerical solutions of the HJE characteristic equations in the construction of the WKB approximation.

To conclude, the book presents original and cutting-edge methods in asymptotics (for PDEs and stochastic processes) used to resolve classical and new questions in filtering, synchronization, and tracking theory. It presents algorithms for the analysis of tracking systems that are not only relevant in communications theory, but also can be used to analyze synchrony in biological and other systems.

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Stochastic Processes: From Physics to Finance. Second Edition. *By W. Paul and J. Baschnagel.* Springer, New York, 2013. \$109.00. xiv+280 pp., hardcover. ISBN 978-3-319-00326-9.

The authors, both physicists, have revised their successful book first published in 2000. It was based on a seminar that they held at the University of Mainz on the uses of methods from physics in particular statistical physics, in finance mathematics, which is now often called econophysics. The emphasis is on ideas and techniques with examples and explanations rather than proofs, but the stochastic processes are presented clearly in mathematical language, e.g., with measure theoretical formalism, which makes the book readable for mathematicians. Its value for mathematicians, especially those who are already familiar with the basic ideas of mathematical finance, is in the many examples from physics, that provide a broad overview of the basic models and ideas of statistical physics.

The new edition contains additional as well as revised material. In Chapter 2, Jaynes's treatment of probability as a form of logic is used to judge rational expectations and to introduce his maximum entropy principle. In addition, there is a discussion on distributions of extreme values. Chapter 3 now includes a section on the Caldeira–Leggett model, which allows a generalized Langevin equation to be derived from deterministic Newtonian mechanics. There is also a section on first passage times for unbounded diffusions as an example of the effectiveness of renewal equation techniques and a discussion on extreme excursions of Brownian motions. In addition, the section on Nelson's stochastic mechanics has been extended to provide a detailed discussion on the tunneling effect. Much of the material on credit risk analysis in Chapter 5 was made obsolete by the financial crisis in 2008 and has been appropriately modified. A major new development is the treatment of nonstationarity of financial time series, with additional discussion of extreme events in such series. Finally, the treatment of microscopic modeling approaches has been extended to include agent-based modeling techniques, which allows correlation of agent behavior and microscopic degrees of freedom to be incorporated in the discussions. There are six appendices that expand on background mathematical material.

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Nonlinear Time Scale Systems in Standard and Nonstandard Forms: Analysis and Control. *By Anshu Narang-Siddarth and John Valasek*. SIAM, Philadelphia, 2014. $94.00. xvi+219 pp., hardcover. ISBN 978-1-611973-33-4.

Singular perturbation methods in control have provided a very successful application of many asymptotic techniques, involving applied mathematicians and engineers. The early work was summarized in Kokotovic, Khalil, and O'Reilly [1], now reprinted as a SIAM Classic. This new book has developed from the recent thesis research of Professor Narang-Siddarth at Texas A&M University, where Professor Valasek was her advisor. It is less specifically oriented toward aerospace applications than Ramnath [2] and indeed, it begins with a presentation of multiple time scale phenomena quite generally before considering design aspects and stabilizing controls.

The standard problem consists of the initial value problem for the coupled slow-fast vector system

$$\dot{x} = f(t, x, z, u),$$
$$\epsilon\dot{z} = g(t, x, z, u),$$

with a small positive parameter ϵ. Its limiting outer solution away from a thin initial layer results when we can solve the limiting algebraic constraint for

$$z = h(t, x, u),$$

resulting in a reduced-order control problem. These authors call the problem nonstandard when they can't solve for z in this way. They naturally seek ways to transform the given problem to a standard one. The control aspect makes the problem interesting, and computed solutions for aerospace and other examples provide a check on any intuitive design choices made.

Not surprisingly, inner and outer (or slow and fast) problems arise, and one naturally seeks the composite control as the sum of slow and fast parts. Stability hypotheses naturally involve Liapunov functions, and extensions with a hierarchy of several small parameters multiplying derivatives occur. There's a nice overview of classical results, including an emphasis on a role of the slow manifold. Most significant and novel, however, is the treatment of nonstandard examples. This is very worthy of further development, regarding both theory and practice. The authors deserve our thanks for their successful and provocative developments.

REFERENCES

[1] P. KOKOTOVIC, H. K. KHALIL, AND J. O'REILLY, *Singular Perturbation Methods in Control: Analysis and Design*, Academic Press, London, 1986.

[2] R. V. Ramnath, *Multiple Scales Theory and Aerospace Applications*, AIAA Education Series, Reston, VA, 2010.

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Ordinary Differential Equations, from Calculus to Dynamical Systems. *By Virginia W. Noonburg.* The Mathematical Association of America, Washington, D.C., 2014. $60.00. xiv+315 pp., hardcover. ISBN 978-1-93951-204-8.

This book has the traditional outline of a first course in ODEs: Introduction, first-order equations, second-order equations, linear systems, geometry of autonomous systems, and Laplace transforms. Overall, there are lots of pictures of solutions. Students are encouraged to use computer algebra and numerical methods. Examples (and projects) coming from easy-to-comprehend applications are common, and complicated solution techniques aren't avoided when needed. Readers, in keeping up, will learn a lot that will be useful elsewhere.

There's particularly good coverage of beats and resonance, phase plane pictures, the matrix exponential (and its simplicity compared to corresponding eigenvalue/eigenvector representations), bifurcation, limit cycles, and the Laplace transform (which many authors make so simple that it provides no added value).

The writing is clear, the problems are good, and the material is well motivated and largely self-contained. Some previous acquaintance with linear algebra would, however, be helpful.

In summary, this new book is highly recommended for students anxious to discover new techniques.

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Differential Equations and Linear Algebra. *By Gilbert Strang.* Wellesley-Cambridge Press, Wellesley, MA, 2014. $86.00. x+502 pp., hardcover. ISBN 978-0-9802327-9-0.

There's no doubt that Gilbert Strang is a master teacher and an enthusiastic evangelist for his perceptive vision of where applied math should be headed. After a half century and ten editions of Boyce and DiPrima, there's a pile of reasons (and ways) to change the typical first course in differential equations. One good idea is to combine that course with one on linear algebra, which occurred quite some time ago to Kreider, Kuller, Ostberg, and Perkins and to Hirsch and Smale, among others. Now, however, we have MATLAB and Maple, the singular value decomposition, and the fast Fourier transform! Some experimentation with technology and computing uncovers the practical importance of differential equations. Students tend to learn the method of Frobenius and about specific special functions later, perhaps encountering them in a course in engineering, biology, or finance. They ultimately also learn that nonlinearity must be faced. This is hinted at by the book's attractive cover illustration (by two artistic SIAM staff members), which relates pictures of the Lorenz attractor from a Portuguese grad student.

As you'd expect, the emphasis here is linear differential equations with constant coefficients. Honestly, there aren't many variable coefficient ODEs that we can handle analytically, though it is certainly fun to solve one. Numerical methods for initial value problems are, admittedly, very successful and the resulting portraits provide immediate understanding of solution behavior. Moreover, the powerful underlying A^TA philosophy employed carries over to using eigenvalues and eigenvectors to solve boundary value problems for Laplace's equation and other partial differential equations, analytically and via finite differences. Most sophomores would not have realized this without Strang's insistence. Using Fourier series and Fourier and Laplace transforms brings the focus successfully back to the classical syllabus. Meanwhile, however, one has figured out many matrix decompositions, how to use delta and transfer functions, and has understood critical ideas like stability and stiffness. The exercises, which include challenge problems, look interesting, and extensive backup resources from MIT websites are available.

As with Strang's linear algebra books, now in their fourth edition, this text is destined to have a big impact on differential equations courses and applied math education. Its conversational presentation, breadth, and provocative problems will even appeal to students, who typically read little of the book assigned. Those who teach differential equations should definitely give Strang's approach serious consideration. Once again, he's making us think!

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Brownian Dynamics at Boundaries and Interfaces: In Physics, Chemistry, and Biology. *By Zeev Schuss.* Springer, New York, 2013. $79.99. xx+322 pp., hardcover. ISBN 978-1-4614-7686-3.

The random movement of ions and molecules caused by collisions between them is known as Brownian motion. Traditionally, the mathematical theory of Brownian motion has been used to describe chemical kinetics and the transport of molecules in physical systems. However, chemical kinetics is present everywhere in biological systems. For instance, many physiological processes relate to the diffusion of calcium ions and the activation caused by the subsequent binding of the ions to receptors. This book uniquely combines an introduction to the mathematical theory of Brownian motion with applications to chemical kinetics, primarily in biology and physiology.

The author makes a special effort to emphasize the difference between two distinct notions of Brownian motion. "Mathematical Brownian motion" is a Wiener process, $W(t)$, that can be thought of as a random walk for which steps in time and space are infinitesimally small. "Physical Brownian motion" refers to the Langevin description of a particle in which the displacement of the particle, call it $x(t)$, satisfies an equation of the form $m\ddot{x}(t) + \lambda\dot{x}(t) = \dot{W}(t)$.

In the limit of infinite damping the Langevin description becomes a Wiener process, and so in certain situations a Wiener process provides a useful approximation to the motion of a Langevin particle. Mathematically, this approximation is convenient as it reduces the number of dimensions that we have to work with by one. However, there are undesirable physical consequences. In particular, suppose a Wiener process crosses a threshold value at some time T. Then the process also crosses the threshold value at infinitely many other times within any open time interval containing T (with probability 1). This contrasts with how we expect particles to behave and has important consequences in biological applications where molecules cross membranes or hit the boundary of some component of a neuron.

The first half of the book concerns essential mathematical theory. This centers around stochastic differential equations. The book assumes no prior knowledge of stochastic differential equations and so begins by introducing Wiener processes, stochastic integrals, and Ito's formula, before progressing to the Fokker–Planck equation, numerical methods, and various aspects of first passage problems. The reader will require background in ODEs, PDEs, and statistical theory. Relative to other textbooks on stochastic differential equations, a large amount of material is covered in a short space. For this reason, on one hand the first half of the book constitutes a wonderfully useful reference, but on the other hand I feel it would be somewhat hard for a novice to learn from, particularly as the very first section is technical and heavily notational.

The second half of the book concerns applications. Stochastic differential equations have long been applied to problems in finance, but here the theory is applied to problems that will be familiar to applied mathematicians working in mathematical biology. For instance, between neurons, molecules look to bind to receptors that are spread sparsely across a surface. Mathematically, this is a problem of "narrow escape" (of the molecules to the receptors) and the author devotes two out of the eight chapters of the book to this topic. Another application is a chemical reaction in which two reactants originate in different compartments of a vessel. In the book it is carefully shown how the reaction rate can be interpreted as the principal eigenvalue of a first passage problem.

Overall, this a unique and valuable book. It has been written with diligence and it is a pleasure to see that it appears to have been carefully edited. The writing is accurate and highly detailed, but is perhaps somewhat terse. Exercises are included throughout the book, particularly relating to the mathematical theory. The book will be extremely useful to both mathematicians and biologists/physiologists, etc., who work at the interface of these two subjects.

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Counterflows: Paradoxical Fluid Mechanics Phenomena. *By V. Shtern.* Cambridge University Press, Cambridge, UK, 2012. $135.00. xiv+470 pp., hardcover. ISBN 978-1-107-02759-6.

Despite great advances in analysis and in computational power, understanding even simple fluid flow remains a challenge, and accurate description of complex fluid flow is invariably difficult and, for many flows, still impossible. Fluid mechanics is one of the most complex and fascinating of physical sciences, dealing with structures that are subtle, intricate, beautiful, and elegant in appearance and regularly visible to us on a great range of scales. It is a pleasure to welcome a new book that seeks to shed light on structures within complex laminar fluid flows.

Why are fluid flows so complex and so difficult to describe, or, as the title suggests, paradoxical? Consider a cubic meter of water, 10^3 kgm, or a little over 50 kMol. It will contain roughly 3×10^{28} water molecules; suppose we approximate this as 10^{27} molecules and imagine them frozen in place in a regular lattice for just an instant in time. Picture this lattice of water molecules and imagine a rule placed along one edge. There would be roughly 10^9 molecules set along the rule, so an intermolecular spacing of order 10^{-9}m. At that length scale, you would see individual molecules, but now start to increase the length scale and consider when will you cease to "see" individual molecules; perhaps when increasing by two orders of magnitude you might still discern individual molecules, but increase by three orders of magnitude and all you will see is a continuum. Thus, the cubic meter of water might be approximately modeled as a continuum at 10^{-6}m, some six orders of magnitude smaller than the original scale. In larger scale applications, for example, an oceanographic simulation, this would be nine or more orders of magnitude smaller than the scale of interest. To compound the complexity, dynamic structures which originate at a molecular scale must propagate through nine or more orders of magnitude of length scales to produce structures at a global scale: in a fluid, neighbors are transitory partners (particles starting close together are not constrained to remain neighbors, nor are flow structures at any scale so constrained), so that structures at the small scales do not necessarily propagate to larger scales in a unique way, leading to the possibility of multiple, but different, flow structures for the same apparent global conditions.

This is one of the key features of fluid mechanics: we can impose symmetry or structure at a global level, but pass down to the smallest scale where we do not have a continuum and that symmetry or structure does not exist; inherent asymmetry at the finest scales can feed back through a long chain of length scales to produce flows that contrast with or appear paradoxical compared to any imposed global symmetry or structure. The really surprising outcome is that for many situations, the fine scale asymmetric motions pass through the length scales in such a way that asymmetry is lost and symmetry restored, so that, remarkably, there may be a unique or only a finite number of flow outcomes at a global level despite the underlying asymmetric structures starting at the molecular scale. It is the case that in most practical situations, with a high ratio of inertia to viscous forces, nonlinear interactions between different flow scales are so intense that there is a multiplicity of flow outcomes at the global scale, reflecting the existence of asymmetries on small scales, and we describe the flow as turbulent. This range of scales shows just how computationally difficult simulation will be when fine scale interactions are the source of global scale structures. This book deals with members of the class of flows which are well ordered or structured on a global scale, and so laminar, but where fluid motion is in some sense not the straightforward inflow here and outflow there. Some of these flows have unique outcomes, but also have counterintuitive structures, while others fall into the class where there can be multiple (but only a few) outcomes on the global scale; all are remarkable and interesting fluid flows. The focus of the book is a demonstration of how analysis (usually some variant of a similarity formulation) can still play a role in our understanding of complex fluid flows.

The book is, after the first introductory chapter, organized into thirteen further chapters. The starting point is conical jets, for which notation for a transformed framework is set out with some examples. From these relatively simple flows, the subsequent chapters move on to flows with swirl, with examples from both experimental observations and analysis. Swirl adds a layer of complexity and interest, and the examples considered bring into play ideas from bifurcation theory and stability theory and range from swirling jets above a plane and application to tornadoes, whirlpools, and cosmic jets, with clear modeling, through to reduction to lower-order systems of equations and asymptotic analysis, used to bring out features in these flows.

The next set of flows are related to swirl in cylindrical devices. Again, the framework is well set out, and experiments and analysis go hand-in-hand in describing how these flows break down, which provides an excellent overview of them.

The scene then moves to the most commonly observed "counterflow," that where separation occurs. The focus of this section begins with Jeffrey Hamel flow and then moves across a whole range of interesting plane flows including spiral vortices, their stability, and various jet-like flows before turning to spatially conical flows and interesting experiments related to Marangoni flows, with a continuous interplay between observation and analysis. This is followed by a short chapter on modeling temperature distribution in conical similarity jets.

Having introduced temperature as an additional flow variable, the author moves to buoyancy effects in conical jets where, as in previous chapters, the emphasis is on considering flows where analysis can enhance understanding. Here the flows considered range from simple buoyant jets, to a model for free convection near a volcano, to convection in a perfect gas.

One experimental effect observed in rotating flows is the formation of an internal recirculation "bubble" about the axis of rotation and, at higher rotation rates, the breakdown of this single recirculation, or vortex, into more complicated structures. This provides material for a tremendously interesting chapter showing not only many experimental results but also how modeling can help with control of vortex breakdown in these flows.

In the penultimate chapter the author turns to flows where magnetic effects can be included, and again provides a good introduction to this area and gives a brief framework for flows where modeling is possible.

The final chapter, on stability of conical flows, might very well have more naturally appeared earlier in the book, but even though it stands a little alone at the end, it is a readable and good introduction to this topic.

This book is a fine addition to the literature on fluid dynamics. The emphasis on examining flows for which we are able to carry out some theoretical work means that the majority of flows considered belong to a subset where the analysis can be set in a similarity framework. Hence, there are many aspects relating to separation that are not considered, particularly when boundary layers become important or when multiple spatial structures are needed to adequately understand, for example, the boundary layer structure at a separation point on a wall; nevertheless, the weaving of experimental description with analysis is well worth reading. This book has material that should interest most fluid dynamicists and should be an accessible source of examples against which numerical simulations can be tested for those involved in computing fluid flows.

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Principles of Multiscale Modeling. *By Weinan E.* Cambridge University Press, New York, 2011. $80.00. xviii+466 pp., hardcover, ISBN 978-1107-09654-7.

Classical mathematical models employing continuum mechanics and differential equations have had a long history of successfully describing macroscopic properties in elementary physical systems. However, problems where such basic models are insufficient are also ubiquitous. A few types of problems sensitive to structural details on smaller scales can be solved with techniques such as matched asymptotic expansions and homogenization theory, but these approaches have limitations. For many modern problems in chemistry, physics, materials science, and other emerging areas, strong coupling between effects at different scales ("multiscale"), with each scale potentially needing fundamentally different types of models ("multiphysics"), has been a longstanding challenge and calls for a different overall modeling approach.

The author of the current book is a leading researcher who has a track record of strong contributions to numerical analysis, fluid dynamics, partial differential equations, and other areas. He was also a member of the founding editorial board of SIAM's journal *Multiscale Modeling & Simulation*, and his book now gives an authoritative introduction and overview of key methods in this very active and growing field.

The preface gives a clear and accessible overview of the hierarchy of physical models, starting from quantum mechanics at the bottom scale, then moving up to molecular dynamics, kinetic models, and finally continuum mechanics at the largest scales. The author also emphasizes that development of effective computational methods is essential in making advances on multiscale problems. Reflecting this, the author's classification of problems into "Type A" (with spatially isolated microscale features) and "Type B" (with microscale effects distributed throughout) occurs as part of his description of how computational strategies for these problems differ. While a sizable portion of the book focuses on numerical methods, this is not a book on numerical analysis. The presentation is primarily aimed at building up to the current state of the art in designs for robust and efficient multiscale scientific computing.

Before the new computation strategies are addressed in full, several chapters establish the background for readers. Chapter 2 gives a self-described "crash course" in classical and modern analytical methods for multiscale problems. This includes matched asymptotics, averaging, WKB, homogenization, and multiscale expansions on the classical side, with renormalization group analysis, stochastic simulation algorithms, and the Mori–Zwanzig formalism for modern methods. The author has set a good balance between "precision" and "accessibility" that makes the material concise but still captures the essential elements. Good illustrative examples are briefly presented and, as in later chapters, extensive background references are provided for readers wishing to study any of these topics in greater detail.

Chapter 4 similarly gives a concise presentation of the four fundamental levels of physical models mentioned in the preface (continuum, molecular, kinetic, and quantum descriptions). The author highlights that the smaller-scale physics is the direction needing further development. Consequently, quantum mechanics is given the most comprehensive review, covering the classic tight binding approximation through to Kohn–Sham density functional theory.

Analogous to Chapter 2 but on the computational side, Chapter 3 provides an accelerated overview of classical multiscale numerical methods. These methods, including multigrid, domain decomposition, adaptive mesh refinement, and the fast multipole method, are limited by the linear growth of computational workload with the resolution of the finest-scale structures. The modern methods that the book ultimately presents circumvent this by exploiting separation of scales to efficiently solve different physical models appropriate to each scale and developing good strategies for coupling and exchanging information between scales.

Chapter 5 sets the stage for the later, more computationally intensive multiscale problems by first giving examples where multiscale analysis can yield improved continuum models for Type A and B problems. In particular, three fundamental problems where molecular dynamics affect the continuum models are discussed: constitutive properties of polymeric fluids, nonlinear elasticity of solids, and moving contact lines at the edges of spreading fluids. The next two chapters address the current methods for computing Type B and A problems, respectively. The detailed presentation of the approach developed by the author, the heterogeneous multiscale method (HMM), and how to make coupling between scales "seamless," make Chapter 6 the heart of the book. The chapter is also enriched by its discussion of how the computational strategy underlying the HMM compares against other current approaches (extended multigrid and equation-free methods).

Domain decomposition and other adaptive multiscale computational methods for Type A problems are illustrated in Chapter 7 in the context of three physical systems (shocks in gas dynamics and the problems of defects in solids and motion of contact lines in fluids that were introduced earlier). The following three chapters develop aspects of the HMM further for other classes of problems. Chapter 8 compares the HMM to other forms of multiscale models for finite element methods for elliptic partial differential equations. Chapter 9 focuses on modeling of evolutionary problems having multiple temporal scales: stiff ODE systems and stochastic simulations for chemical reactions and epidemiological models. Further aspects of stochastic processes are examined in the context of rare events and transition state theory in Chapter 10.

The book is a sophisticated introduction to the field and expects readers to arrive with a solid background in many of the ingredients that are involved in constructing multiscale models; nevertheless, it provides a very valuable perspective on current methods. While the classes of problems addressed cover only a particular range of applications, the book helps to bridge the gap between classic analytical models and traditional numerical methods and the new approaches needed for the current challenging open problems. Factoring out its length, I'd compare its presentation of multiscale modeling with the style of and comprehensive overview given in some of the best survey articles of other fields appearing in *SIAM Review*.

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