1.

$$\frac{d^4U(x,s)}{dx^4}=\delta(x-s)$$

use Fourier translation and we can get

$$(ik)^4\overline{U}(k,s)=e^{-iks}\quad\Rightarrow\quad\overline{U}(k,s)=\frac{1}{k^4}e^{-iks}$$

use inverse Fourier translation

$$U(x,s)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{k^4}e^{ik(x-s)}dk\Rightarrow\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{z^4}e^{iz(x-s)}dz$$

expand $e^{iz(x-s)}$ in Taylor series

$$e^{iz(x-s)}=1+i(x-s)z-\frac{1}{2}(x-s)^2z^2-\frac{i}{6}(x-s)^3z^3+\cdots$$

When $x>s$

$$\oint\frac{1}{z^4}e^{iz(x-s)}dz=\oint\left(\frac{1}{z^4}+i(x-s)\frac{1}{z^3}-\frac{1}{2}(x-s)^2\frac{1}{z^2}-\frac{i}{6}(x-s)^3\frac{1}{z}\right)dz=0$$

$$\int_{c_1+c_2}dz+\int_{c_R}dz+\int_{c_\rho}\left(\frac{1}{z^4}+i(x-s)\frac{1}{z^3}-\frac{1}{2}(x-s)^2\frac{1}{z^2}-\frac{i}{6}(x-s)^3\frac{1}{z}\right)dz=0$$

Let $z=\rho e^{i\theta}$, $dz=i\rho e^{i\theta}d\theta$

$$\int_{\pi}^{0}\left(\frac{1}{z^4}+i(x-s)\frac{1}{z^3}-\frac{1}{2}(x-s)^2\frac{1}{z^2}-\frac{i}{6}(x-s)^3\frac{1}{z}\right)dz$$

$$=\int_{\pi}^{0}\left(\frac{1}{\left(\rho e^{i\theta}\right)^4}+i(x-s)\frac{1}{\left(\rho e^{i\theta}\right)^3}-\frac{1}{2}(x-s)^2\frac{1}{\left(\rho e^{i\theta}\right)^2}-\frac{i}{6}(x-s)^3\frac{1}{\rho e^{i\theta}}\right)i\rho e^{i\theta}d\theta$$

$$=-\frac{2}{3\rho^3}+0+\frac{(x-s)^2}{\rho}-\frac{\pi(x-s)^3}{6}$$

$$\frac{1}{2\pi}\left[\int_{c_1+c_2}dz-\frac{2}{3\rho^3}+0+\frac{(x-s)^2}{\rho}-\frac{\pi(x-s)^3}{6}\right]=0$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{z^4}e^{iz(x-s)}dz=\frac{1}{2\pi}\left[\int_{c_1+c_2}dz-\frac{2}{3\rho^3}+0+\frac{(x-s)^2}{\rho}\right]=\frac{(x-s)^3}{12}$$

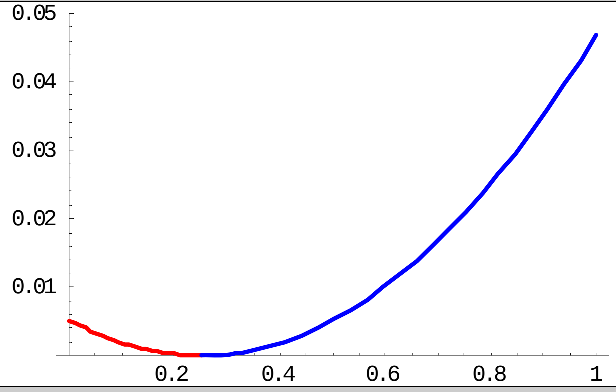
$$U(x,s)=\frac{(x-s)^3}{12},\quad x>s$$

similarly

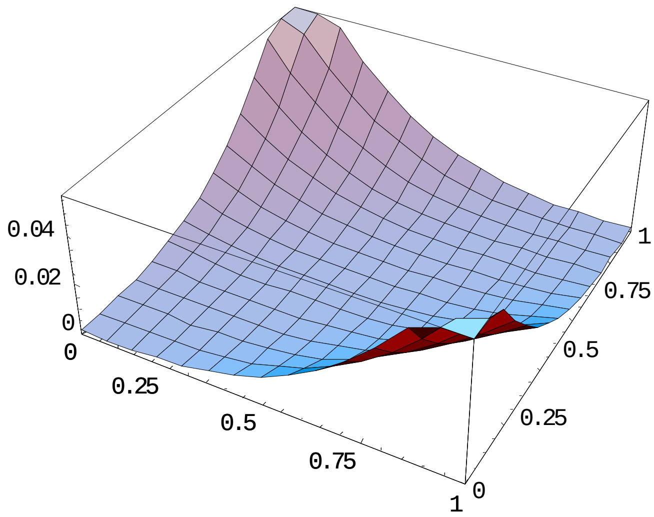
$$U(x,s)=\frac{(s-x)^3}{12},\quad s>x$$

$U(x,s)$ is regular, symmetric and degenerate form.

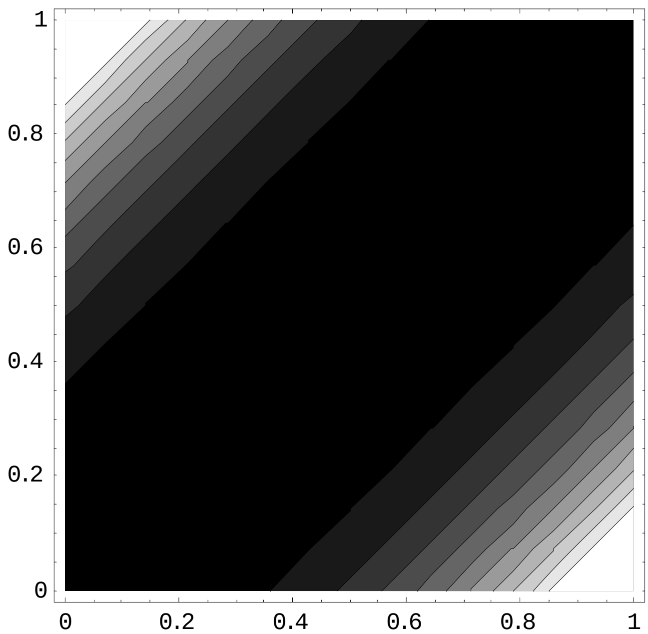
2-D



3-D



Contour



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