problem. It may be greater near the center of an element than near a node. As a rule of thumb a degradation of accuracy may occur at distances less than one element length from a boundary. Thus if greater accuracy is required, more shorter elements can be used.

In porous media problems the velocities are often needed as well as the head (potential). The velocities are related to the derivatives of the potential as in (1.7):

$$q_x = -K\frac{\partial \Phi}{\partial x} \qquad q_y = -K\frac{\partial \Phi}{\partial y} \tag{2.66}$$

The derivatives of Φ can be found by differentiating (2.13):

$$2\pi\frac{\partial}{\partial x}\Phi(P) = \int_\Gamma \left[\Phi(Q)\frac{\partial}{\partial x}\left(\frac{1}{r}\frac{\partial r}{\partial n}\right) - \frac{\partial}{\partial x}(\ln r)\frac{\partial}{\partial n}\Phi(Q)\right] ds \tag{2.67}$$

Only Φ at the base point P is differentiated since that is where we want to find the derivative. The field point Q is on the boundary and thus the Φ inside the integral is not differentiated. We now use $\partial r/\partial n = \eta/r$ and obtain

$$2\pi\frac{\partial \Phi}{\partial x} = \int_\Gamma \left(\frac{\Phi}{r^2}\frac{\partial \eta}{\partial x} - 2\frac{\Phi}{r^3}\eta\frac{\partial r}{\partial x} - \frac{1}{r}\frac{\partial r}{\partial x}\frac{\partial \Phi}{\partial n}\right) ds \tag{2.68}$$

Consider the integration from a base point P along an element extending from ξ_j to ξ_{j+1} as shown in Figure 2.7. At the point $Q(\xi, \eta)$

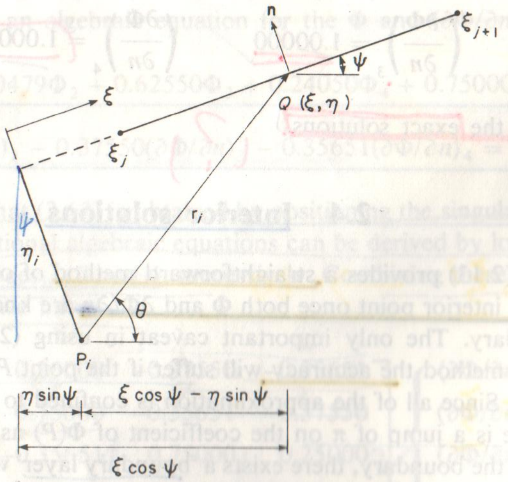

Figure 2.7 Definition sketch for the interior solution.

$$\frac{\partial \eta}{\partial x} = \sin\psi \tag{2.69}$$

$$\frac{\partial r}{\partial x} = -\frac{\xi\cos\psi - \eta\sin\psi}{r} \tag{2.70}$$

Using these values in (2.68) produces

$$\frac{\partial}{\partial x}\Phi(P) = \frac{1}{2\pi}\int_\Gamma \left[\frac{\Phi\sin\psi}{r^2} + \frac{2\Phi\eta(\xi\cos\psi - \eta\sin\psi)}{r^4} + \frac{\xi\cos\psi - \eta\sin\psi}{r^2}\frac{\partial\Phi}{\partial n}\right] ds \tag{2.71}$$

Similarly,

$$\frac{\partial}{\partial y}\Phi(P) = \frac{1}{2\pi}\int_\Gamma \left[-\frac{\Phi\cos\psi}{r^2} + \frac{2\Phi\eta(\eta\cos\psi + \xi\sin\psi)}{r^4} + \frac{\eta\cos\psi + \xi\sin\psi}{r^2}\frac{\partial\Phi}{\partial n}\right] ds \tag{2.72}$$

Unlike (2.13) the point P cannot be moved to the boundary to find the derivatives. The singularities that occur when the base point and field point coincide are not integrable. in his sense

The integrals involved in (2.71) and (2.72) are shown in Appendix A.4. In solving all the above integral equations care should be taken to obtain the proper sign on η. If η and **n** are in the same direction as shown in Figures 2.5 and 2.7, η is positive. If η and **n** are in opposite directions, as may be the case if the vector **r** from P to the point Q passes outside of the region, the sign on η is negative.

$G(\underset{\sim}{x}, \underset{\sim}{\xi}') \rightarrow \infty$

$\underset{\sim}{x} = \underset{\sim}{\xi}$