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The application of boundary element evaluation on a silencer in the presence of a linear temperature gradient

Chao-Nan Wang *, Yih-Nan Chen, Jean-Yih Tsai

Department of Naval Architecture and Ocean Engineering, National Taiwan University, Taipei, Taiwan, ROC

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Abstract

A boundary element method for analyzing the acoustic performance of a muffler in the presence of a linear temperature gradient is developed. In order to simulate the temperature gradient, the muffler is divided into segments with different constant temperatures. The boundary element approach is applied to each sub-volume to establish the relationship between pressure and its gradient on the surface. Then combining all the sub-volumes by matching the continuity of pressure and velocity on the contact surface to obtain the relationship of the inlet and outlet of a muffler Consequently, the transmission loss of a muffler can be evaluated. In the present study, the performance of mufflers with different temperature and temperature gradients is investigated.

1. Introduction

It is well known that the muffler is a widely used apparatus in noise control. In order to design a muffler, the understanding of sound propagation in it is very important. Many workers [1–5] have carried out analysis of mufflers, such as, the simple expansion chambers, straight-through resonators and plug mufflers, etc. Their studies not only considered the effect of perforation but also included the effect of mean flow. Most of the analyses gave good results. The temperature, however, is an another important factor and needed to be considered. Prasad and

* Corresponding author.

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Crocker [6] and Munjal and Prasad [7] applied methods to evaluate the four-pole parameters for a plane-wave propagated in a uniform pipe in the presence of a mean flow and a temperature gradient, respectively. Peat [8] proposed some inconsistency in Ref. [6] and also gave a simpler perturbation method to evaluate the transfer matrix of a uniform duct. Kim et al. [9] also derived a recursive theoretical formulation to solve the acoustic characteristics of a circular expansion chamber for which higher order modes existed.

In this study, the governing equation for acoustic wave propagation through an expansion chamber with a linear temperature gradient is derived. Then the expansion chamber is divided into M segments and the temperature is assumed to be constant. The relationship of pressure and its gradient for a muffler is obtained by applying boundary element analysis to each segment and matching the pressure and velocity continuity condition between the adjacent segments. Hence the performance of a muffler can be evaluated by applying suitable boundary conditions.

2. Theoretical formulation

The silencer to be considered is a simple expansion chamber as shown in Fig. 1. Assuming that the temperature decreases linearly along the silencer from T_1 to T_2 at inlet and outlet respectively. The sound speed $c(x)$ and mean density $\rho_0(x)$ of the medium are changed due to the temperature variation. The three-dimensional acoustic waves propagated in the expansion chamber of the mean flow velocity V should be governed by [9]

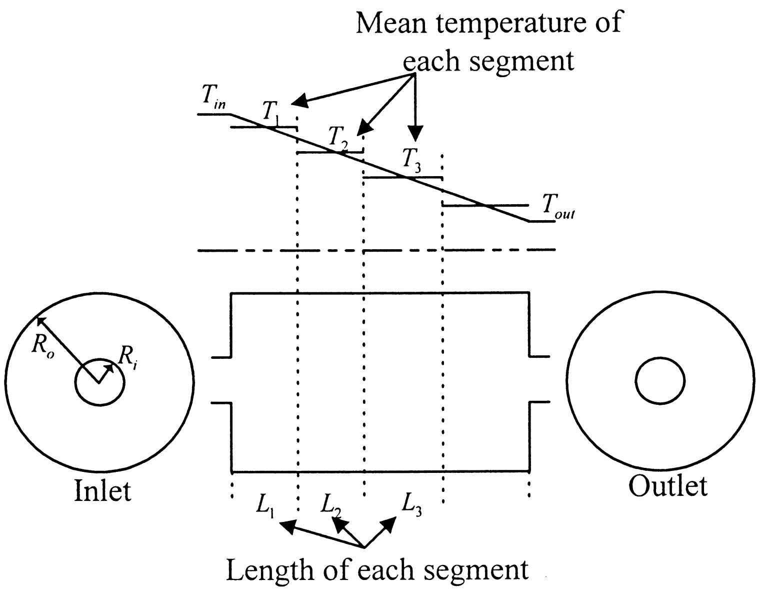

Fig. 1. The geometry of a simple expansion silencer in the presence of linear temperature gradient.

$$\nabla^2\phi + k^2(x)\phi - 2ik(x)M(x)\frac{\partial\phi}{\partial x} - M(x)^2\frac{\partial^2\phi}{\partial x^2} = 0 \tag{1}$$

where wave number $k(x) = \omega/c(x)$, Mach number $M(x) = V/c(x)$ and $\phi(x)$ is the velocity potential. Since the wave number is a function of x, the solution becomes more complicated. To simplify the problem, the silencer is divided into N cylindrical segments and each of them has a constant temperature. Thus for each sub-volume the acoustic wave is governed by

$$\nabla^2\phi + k^2\phi - 2ikM\frac{\partial\phi}{\partial x} - M^2\frac{\partial^2\phi}{\partial x^2} = 0 \tag{2}$$

By applying the Prandtl–Glauert transformation [10]

$$\tilde{x} = \frac{x}{\sqrt{1-M^2}}, \quad \tilde{y} = y, \quad \tilde{z} = z \tag{3}$$

and let

$$\tilde{\phi} = \phi e^{-i\xi\tilde{x}}, \tag{4}$$

$$\xi = \frac{kM}{\sqrt{1-M^2}}, \tag{5}$$

and

$$\tilde{k} = \frac{\xi}{M} \tag{6}$$

Eq. (2) can be reduced to a Helmholtz equation in the transformed domain

$$\tilde{\nabla}^2\tilde{\phi} + \tilde{k}^2\tilde{\phi} = 0 \tag{7}$$

Applying the Green second identity to each sub-volume and also introducing the fundamental solution of the Helmholtz equation gives the boundary integral equation as

$$C(\beta)\tilde{\phi}(\beta) = \int\int_{\tilde{S}}\left[G(\beta,\alpha)\frac{\partial\tilde{\phi}}{\partial\tilde{n}}(\alpha) - \frac{\partial G(\beta,\alpha)}{\partial\tilde{n}(\alpha)}\tilde{\phi}(\alpha)\right]\mathrm{d}S(\alpha) \tag{8}$$

The fundamental solution and coefficient C are expressed as [10]

$$G(\beta,\alpha) = \frac{e^{-i\tilde{k}\tilde{r}(\beta,\alpha)}}{\tilde{r}(\beta,\alpha)} \tag{9}$$

and

$$C(\beta) = -\int\int_{\tilde{S}} \frac{\partial}{\partial \tilde{n}(\alpha)} \left[\frac{1}{\tilde{r}(\beta, \alpha)} \right] \mathrm{d}S(\alpha) \tag{10}$$

where $\tilde{r}$ is distance between points α and β and $\tilde{n}$ is outward unit normal vector on the transformed domain.

3. Numerical implementation

For each sub-volume, the boundary surface is divided into elements and the numerical integration is applied to the whole elements for each node. Therefore, the equation for the relationships of the transformed boundary nodal velocity potentials and their gradients are obtained. By Eq. (4), it can be converted to real physical domain and we obtain

$$\left[\hat{H}\right]\{\phi\} = \left[\hat{Q}\right]\left\{\frac{\partial \phi}{\partial n}\right\}, \tag{11}$$

where $\left[\hat{H}\right], \left[\hat{Q}\right]$ are the coefficient matrix obtained from the boundary integral equation. The velocity potential and its gradient can also be converted to pressure and normal velocity by

$$p = \rho_0(i\omega\phi) + \rho_0 V \frac{\partial \phi}{\partial x} \tag{12}$$

and

$$u_n = -\frac{\partial \phi}{\partial n} \tag{13}$$

and we obtain

$$[H]\{p\} = [Q]\{\rho_0 c u_n\} \tag{14}$$

If the nodes are grouped in three parts, i.e. inlet, outlet and other, the above equation becomes

$$\begin{Bmatrix} P_{\text{inlet}} \\ P_{\text{outlet}} \\ P_{\text{other}} \end{Bmatrix} = [A] \begin{Bmatrix} \rho_0 c u_{n \text{ inlet}} \\ \rho_0 c u_{n \text{ outlet}} \\ \rho_0 c u_{n \text{ other}} \end{Bmatrix} \tag{15}$$

where $[A] = [H]^{-1}[Q]$. In the present study, the wall is assumed to be rigid. Thus, the pressure and normal velocity at the inlet and outlet for each sub-volume is connected by

$$\begin{Bmatrix} P_{\text{inlet}} \\ P_{\text{outlet}} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{Bmatrix} \rho_0 c u_{n\ \text{inlet}} \\ \rho_0 c u_{n\ \text{outlet}} \end{Bmatrix} \tag{16}$$

Consider two adjacent sub-volumes (e.g. segment i and $i+1$), the matrix equations are, respectively, written as

$$\begin{Bmatrix} p^i_{\text{inlet}} \\ p^i_{\text{interface}} \end{Bmatrix} = \begin{bmatrix} A^i_{11} & A^i_{12} \\ A^i_{21} & A^i_{22} \end{bmatrix} \begin{Bmatrix} \rho_0 c u^i_{n\ \text{inlet}} \\ \rho_0 c u^i_{n\ \text{interface}} \end{Bmatrix} \tag{17}$$

$$\begin{Bmatrix} P^{i+1}_{\text{interface}} \\ P^{i+1}_{\text{outlet}} \end{Bmatrix} = \begin{bmatrix} A^{i+1}_{11} & A^{i+1}_{12} \\ A^{i+1}_{21} & A^{i+1}_{22} \end{bmatrix} \begin{Bmatrix} \rho_0 c u^i_{n\ \text{interface}} \\ \rho_0 c u^{i+1}_{n\ \text{outlet}} \end{Bmatrix} \tag{18}$$

where the interface denotes the contact surface between segment i and $i+1$. On the interface surface, the pressure and the normal velocity must be continuous. Therefore, Eqs. (17) and (18) can be combined to form a transfer matrix for two acoustic segments as

$$\begin{Bmatrix} p^i_{\text{inlet}} \\ p^{i+1}_{\text{outlet}} \end{Bmatrix} = [TF] \begin{Bmatrix} \rho_0 c u^{i+1}_{n\ \text{inlet}} \\ \rho_0 c u^i_{n\ \text{outlet}} \end{Bmatrix} \tag{19}$$

where

$$[TF] = \begin{bmatrix} A^i_{11} - A^i_{12}(A^{i+1}_{11} + A^i_{22})^{-1} A^i_{21} & A^i_{12}(A^{i+1}_{11} + A^i_{22})^{-1} A^{i+1}_{12} \\ A^{i+1}_{21}(A^{i+1}_{11} + A^i_{22})^{-1} A^i_{21} & A^{i+1}_{22} - A^{i+1}_{21}(A^{i+1}_{11} + A^i_{22})^{-1} A^{i+1}_{12} \end{bmatrix}. \tag{20}$$

The transfer matrix of the silencer can be obtained by the series combination of the total acoustic sub-volumes.

4. Transmission loss

All the matrices of each segment are combined as in Eq. (20), then the relationship between the inlet and the outlet of a silencer is expressed as

$$\begin{Bmatrix} p^1_{\text{inlet}} \\ p^N_{\text{outlet}} \end{Bmatrix} = [\Re] \begin{Bmatrix} \rho_0 c u^1_{n\ \text{inlet}} \\ \rho_0 c u^N_{n\ \text{outlet}} \end{Bmatrix} \tag{21}$$

where $[\Re]$ is the total serial combination matrix. The four-pole parameters of the silencer can be evaluated by using two different boundary conditions as follows.

a. Assuming $u^1_{n\ \text{inlet}} = 1$ and $u^1_{n\ \text{outlet}} = 0$, the sound pressure distributions at the inlet and outlet can be obtained and consequently two of the four-pole parameters.

$$A = \frac{P^1_{\text{inlet}}}{P^N_{\text{outlet}}} \quad \text{and} \quad C = -\frac{\rho_0 c u^1_{n\ \text{inlet}}}{p^N_{\text{outlet}}} \tag{22}$$

b. Assuming $p^1_{\text{inlet}} = 1$ and $p^N_{\text{outlet}} = 0$, the normal velocity distributions at the inlet and outlet can be obtained so that

$$B = \frac{p^1_{\text{inlet}}}{\rho_0 c u^N_{n\ \text{outlet}}} \quad \text{and} \quad D = -\frac{\rho_0 c u^1_{n\ \text{inlet}}}{\rho_0 c u^N_{n\ \text{outlet}}} \tag{23}$$

The transmission loss of a silencer can be evaluated in terms of the four-pole parameters by

$$TL = 20\ \log_{10}\left(\frac{A+B+C+D}{2}\right) + 10\ \log_{10}\left(\frac{S_1}{S_2}\right) \tag{24}$$

where S_1 and S_2 denote the cross-sectional area at the inlet and outlet.

5. Numerical simulation

In the present study, the temperature gradient is define as

$$Tr = \frac{T_1 - T_2}{T_1 + T_2} \tag{25}$$

where T_1 and T_2 (K) denote the air temperature at the inlet and outlet, respectively. The effects of temperature gradient and air mean flow velocity on the performance of a silencer are depicted as follows.

5.1. Convergence and verification

The method presented in this study is to divide the silencer into segments with constant temperatures to simulate the temperature gradient. At first, a silencer (length L of 30 cm and diameter D of 15 cm) with temperature gradient $T_r = 0.2$ is adopted to check the convergence of this method. Fig. 2 shows the transmission loss curves of the silencer with different divided segments. It is found that the convergence is rapid and four segments can give a good result. Next, the numerical results are compared to that of an analytical solution [9] to verify the accuracy of the present method. Fig. 3 denotes the transmission loss of a silencer with different temperature gradients ($T_r = 0$, 0.2 and 0.4). When comparing to Fig. 8 of Ref. [9], it is seen that the agreement is good except there is some discrepancy occuring at the higher frequency (above 3200 Hz) for $T_r = 0.4$. It's also found that the curves and the cut-off frequency of the first radial mode extend to the higher frequency range when the temperature gradient increases. Mean flow velocity can also change the performance of a silencer. The same muffler with a different mean flow velocity and constant temperature, as shown in Fig. 4, is also analyzed to illustrate its effect. One finds that the curves shift to lower frequency as the mean flow velocity increases, but the cut-off frequency of the first radial mode only reduces slightly. The results compared to that of analytical solutions (Fig. 7 of Ref. [9]) also agree well.

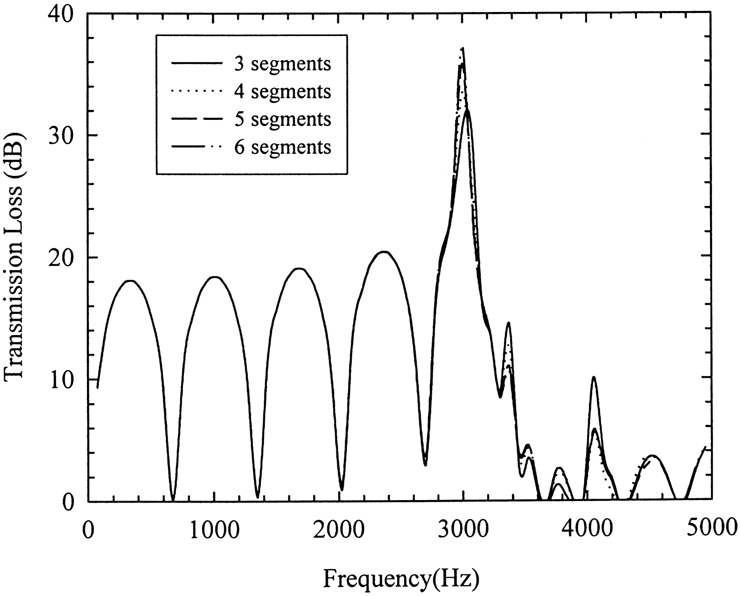

Fig. 2. The convergence of the present method for a silencer in the presence of temperature gradient $T_r = 0.2$.

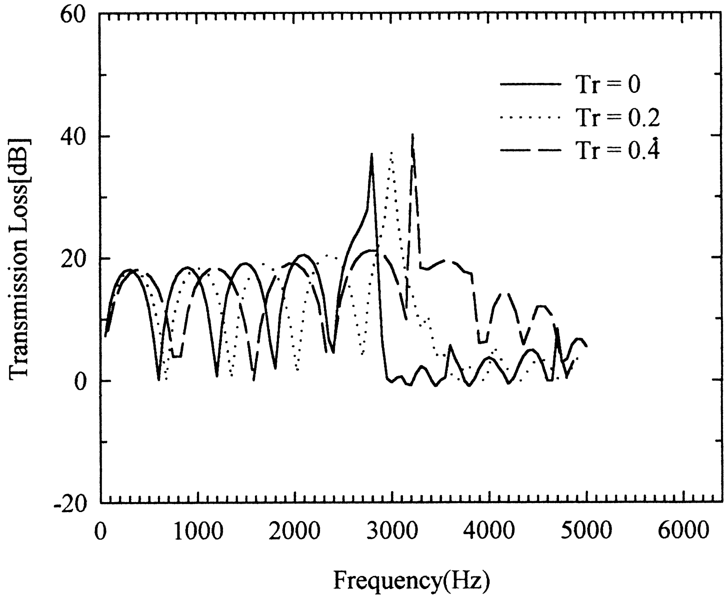

Fig. 3. The transmission loss of a silencer with different temperature gradients.

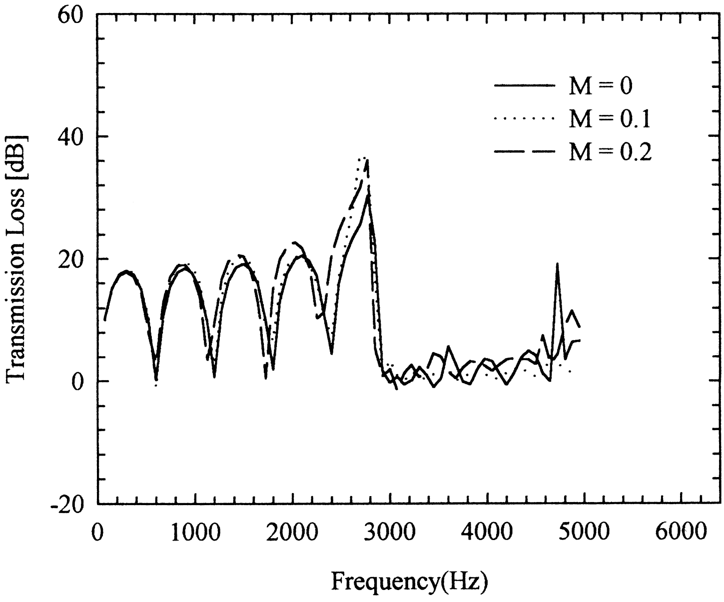

Fig. 4. The transmission loss of a silencer with different mean flow velocities.

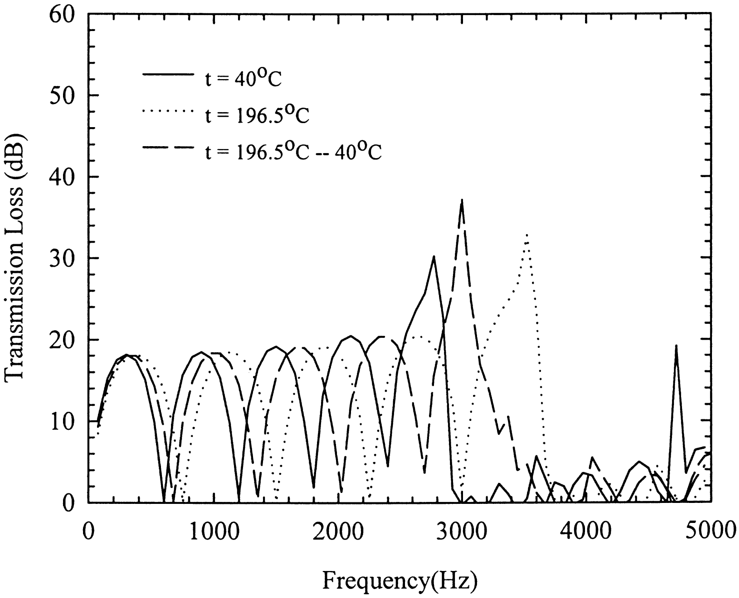

Fig. 5. The effect of constant temperature and temperature gradient on the performance of a muffler.

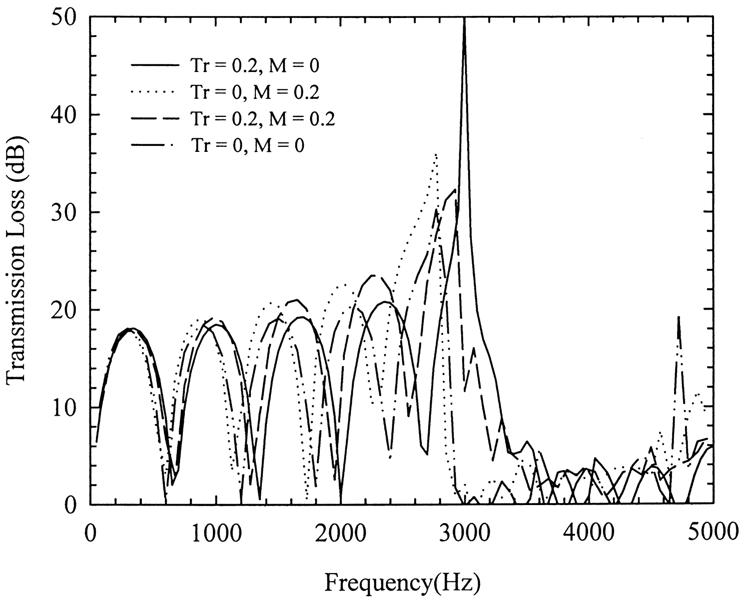

Fig. 6. The combination effect of mean flow velocity and temperature gradient on a silencer.

5.2. The effects of temperature and mean flow

From the result of the previous section, one can see that increasing the temperature will extend the transmission loss curve to a higher frequency. To illustrate more clearly, a muffler with different constant temperature (40 and 196.5°C) and linear temperature gradient $T_r = 0.2$ (i.e. 196.5–40°C) is analyzed and shown in Fig. 5. The cut-off frequency and the transmission loss shift to a higher frequency range due to a temperature increase can be observed easily. Finally, different combinations of mean flow velocity and temperature gradient are analyzed and shown in Fig. 6. The effect of mean flow velocity (shift to low frequency) and temperature gradient (shift to high frequency) is still obvious, individually. The combined case ($T_r = 0.2$ and $M = 0.2$) reveals that temperature gradient seems to give a greater effect on the transmission loss of a silencer.

6. Conclusion

The solution of the acoustic wave propagation in a simple expansion silencer in the presence of mean flow velocity and linear temperature gradient was obtained by the boundary element method. It is found that just a small number of segments is sufficient for convergence. Thus, the extension of the present method to a more general case is feasible. By comparing the numerical results to that of an analytical solution, we found that the present method predicted quite well the transmission loss of a simple expansion chamber. From the numerical result, we also see that the temperature (or its gradient) has a greater effect on the sound propagation in a silencer than mean flow velocity.

References

[1] Ji Z, Ma Q, Zhang Z. Application of the boundary element method to predicting acoustic performance of expansion chamber mufflers with mean flow. Journal of Sound and Vibration 1994;173(1):57–71.

[2] Wang CN, Tse CC, Chen YN. Analysis of three dimensional muffler with boundary element method. Applied Acoustics 1993;40:91–106.

[3] Peat KS. A numerical decoupling analysis of perforated pipe silencer elements. Journal of Sound and Vibration 1988;123(2):199–212.

[4] Munjal ML, Rao KN, Sahasrabudhe AD. Aeroacoustic analysis of perforated muffler components. Journal of Sound and Vibration 1987;114(2):173–88.

[5] Wang CN. A numerical analysis for perforated muffler components with mean flow. ASME, Journal of Vibration and Acoustics 1999;121:231–6.

[6] Prasad MG, Crocker MJ. Evaluation of four-pole parameters for a straight pipe with a mean flow and a linear temperature gradient. J Acoust Soc Am 1981;69:916–21.

[7] Munjal ML, Prasad MG. On plane-wave propagation in a uniform pipe in the presence of a mean flow and a temperature gradient. J Acoust Soc Am 1986;80:1501–6.

[8] Peat KS. The transfer matrix of a uniform duct with a linear temperature gradient. Journal of Sound and Vibration 1988;123:43–53.

[9] Kim YH, Choi JW, Lim BD. Acoustic characteristics of an expansion chamber with constant mass flow and steady temperature gradient (theory and numerical simulation). ASME, Journal of Vibration and Acoustics 1990;112:460–7.

[10] Wang CN, Liao CY. Boundary integral equation method for evaluating the performance of straight-through resonator with mean flow. Journal of Sound and Vibration 1998;216:281–94.