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Wave propagation through arrays of unevenly spaced vertical piles

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Abstract

The interaction of regular waves with arrays of bottom-mounted circular cylinders is considered. This subject has been thoroughly investigated in the recent past, but most of the time under the assumption of regular and spatially periodic arrangements. Unlike these authors, we consider here arrays of unevenly spaced cylinders, displaced randomly from a regular array according to a disorder parameter. Focus is put on two effects of this spacing irregularity: reduction of peak forces associated to trapped mode phenomena, and regularization of the transmission coefficient for waves propagating through the arrays.

Keywords: Potential flow; Gravity wave; Pile arrays; Transmission coefficient; Trapped mode

1. Introduction

Many offshore structures, such as oil rigs, VLFS or some breakwaters (Duclos et al., 2003) are supported by vertical columns. Very large floating structures, supported by hundreds or thousands of piles are now under consideration. So, a fine understanding of the interaction of water waves with such large sets of objects is of fundamental importance to ensure successful design of these structures. Various analytical methods have been developed to solve the wave diffraction problem for arrays of vertical cylinders. The most noticeable early work is due to Twersky (1952) who constructed a solution by using an iterative procedure in which successive scatters by each cylinder were introduced at each order. This method was later

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extended to water-wave case (rather than acoustics) by Ohkusu (1974), while Spring and Monkmeyer (1974) used a direct method to obtain the first-order exiting forces on elements of the arrays by applying the boundary condition on all cylinders simultaneously. This method was further improved by Linton and Evans (1990) who provide simpler expressions for the velocity potential of a particular cylinder. This leads to simple formula for the first and mean second-order forces on the cylinders and also provides an efficient method for the computation of free-surface amplitudes.

A number of recent papers (e.g Maniar and Newman, 1997 and of Evans and Porter, 1997a,b) addressed the possibility that the waves scattered by the columns could constructively interfere, causing large amplitude wave between cylinders and generating very large forces on cylinders, in certain sea conditions, compared to the force on an isolated cylinder in the same sea state. Maniar and Newman worked on the fist-order exciting forces on a large number of identical, equally spaced, circular cylinders in a row. They showed that near-trapped modes occur between adjacent cylinders at some critical wavenumbers, and that this could result in very large loads on individual elements of the array.

Evans and Porter (1997a,b) considered arrays of identical bottom-mounted circular cylinders arranged in a circle. They showed that near-trapped modes can also occur with this kind of arrangement and they demonstrated how the largest forces arise when the near-trapped mode corresponds to a standing wave motion.

In this paper, we consider arrays of identical bottom-mounted cylinders. The general formulation of Linton and Evans, which is recalled in Section 2, is used to determine the velocity potential. In the initial layout, piles are positioned in regular arrays composed of series of the four cylinders cell considered by Evans and Porter (1997a,b). Then, we introduce random disorder in the pile group. The disorder is gradually increased, from very low values, in order to assess its effect on the near-trapped modes observed in regular pile networks. Then, the focus is put on the effect of this arrangement perturbation on wave transmission through the pile groups.

2. Formulation

The scattering of water wave by a group of N bottom-mounted circular cylinders is solved by using the Linton and Evans (1990) formulation; their theory is recalled in this section, keeping their notation for the sake of simplicity. The general framework is the linear theory of free surface potential flow based on the assumption of inviscid fluid and small wave steepness. Under these hypotheses, the velocity potential, solution of Laplace's equation in the fluid domain, can be expressed as

$$\Phi(x, y, z, t) = Re\{\{\phi(x, y)f(z)e^{-i\omega t}\}\} \tag{1}$$

with

$$f(z) = \frac{-igA}{\omega}\frac{\cosh\kappa(z+h)}{\cosh\kappa h} \tag{2}$$

where h is the water depth, g the acceleration of gravity, ω the circular wave frequency, A the incident wave amplitude and κ the wavenumber, real positive solution of the dispersion relation

$$\kappa \tanh \kappa h = \frac{\omega^2}{g} \tag{3}$$

The horizontal potential $\phi(x, y)$ must to satisfy

$$(\nabla^2 + \kappa^2)\phi(x, y) = 0 \tag{4}$$

outside the cylinders, and the impermeability condition

$$\frac{\partial \phi}{\partial n} = 0 \tag{5}$$

on the surface of each of these cylinders. The total potential is written as

$$\phi(x, y) = \phi_{\text{inc}}(x, y) + \sum_{j=1}^{N} \phi_s^j(x, y) \tag{6}$$

where

$$\phi_{\text{inc}}(x, y) = \mathrm{e}^{\mathrm{i}\kappa r \cos(\theta - \theta_{\text{inc}})} = I_k \mathrm{e}^{\mathrm{i}\kappa r_k \cos(\theta_k - \theta_{\text{inc}})} \tag{7}$$

with $I_k = \mathrm{e}^{\mathrm{i}\kappa(x_k \cos\theta_{\text{inc}} + y_k \sin\theta_{\text{inc}})}$, θ_{inc} is the incident wave propagation direction as shown in Fig. 1, (r_k, θ_k) are polar coordinates of the field point (x, y) relative to (x_k, y_k), the center of the kth cylinder. The general form for the scattered potential due to cylinder j is

$$\phi_s^j = \sum_{n=-\infty}^{\infty} A_n^j Z_n H_n(\kappa r_j) \mathrm{e}^{\mathrm{i}n\theta_j} \tag{8}$$

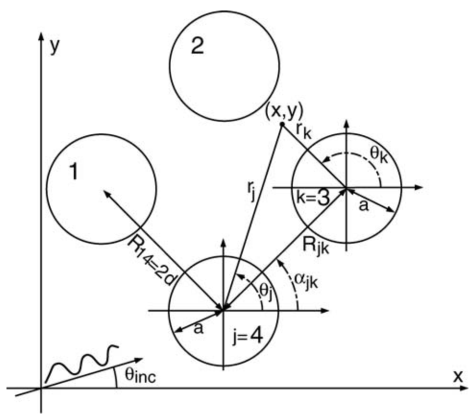

Fig. 1. The circular array of four cylinders.

where $Z_n = J'_n(\kappa a)/H'_n(\kappa a)$, $H_n(\kappa r_j) = J_n(\kappa r_j) + \mathrm{i}Y_n(\kappa r_j)$, J_n and Y_n are the Bessel functions, a is the radius of the cylinders, all considered identical in the present study. By using Graf's addition theorem for Bessel function, Linton and Evans have shown that the satisfaction of the boundary conditions on all the cylinders ($r_j = a$) implies that the coefficients A^k_m should satisfy the following system of equations

$$A^k_m + \sum_{j=1;j\neq k}^{N} \sum_{n=-\infty}^{\infty} A^j_n Z^j_n \mathrm{e}^{\mathrm{i}(n-m)\alpha_{ij}} H_{n-m}(\kappa R_{jk}) = -I_k \mathrm{e}^{\mathrm{i}m(\pi/2-\theta_{inc})}$$
$$k = 1,\ldots,N, \quad -\infty < m < \infty \tag{9}$$

where R_{jk} is the distance between the center of cylinders j and k, and α_{jk} is the angle between the positive x-direction and the line from the center of cylinder j to the center of cylinder k, as shown in Fig. 1. Given the radius a of the cylinders and the position (x_j, y_i) of their center, the linear system Eq. (9) can be assembled and solved for the A^j_n after truncation of the infinite rank m to a suitable value M. In all the computations reported hereafter, $M \leq 7$ was sufficient to obtain an excellent accuracy. Once system Eq. (9) has been solved, the total potential can be recovered, and the wave elevation and wave forces can be straightforwardly computed.

The first-order force on the jth cylinder which is given by integrating the pressure over the surface of the cylinder can be write

$$\left|X^j\right| = \frac{1}{2}\left|F^j\right|\left|A^j_{-1}\{\mp\}A^j_i\right| \tag{10}$$

where

$$F^j = \frac{4\rho g A \tanh \kappa h}{\kappa^2 H'_1(\kappa a)} \tag{11}$$

is the first-order force on an isolated cylinder of radius a in the same incident wave train. (The minus sign in the bracket pair leads to the x-component, and the plus sign to the y-component, respectively.)

3. Perturbation of ordered pile arrangements

The aim of the study was to assess the influence of the level of order of the piles arrangement on certain properties of wave–arrays interaction like near-trapped modes and wave transmission through a large multi rows array. For that purpose, we have first considered arrays of evenly spaced cylinders (circular arrays, matrices of cylinder, rows of cylinders, etc...) with a constant spacing $2d$ between two neighboring cylinders. Fig. 2 shows such a group of 16 piles disposed in a regular three rows arrangement which will serve as an example in the sequel. Then, from each of these regular arrangements of cylinders, we have defined several perturbed arrays by moving at random each cylinder center apart from its original location of

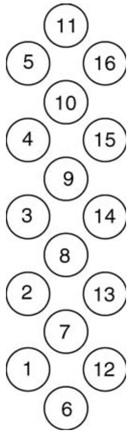

Fig. 2. A three rows ordered arrangement for a 16 piles group.

a displacement defined by $(\Delta x_j, \Delta y_j)$

$$\begin{cases} \Delta x_j = \gamma_j p\tau\cos(2\pi\gamma_j) \\ \Delta y_j = \gamma_j p\tau\sin(2\pi\gamma_j) \end{cases} \tag{12}$$

where the random variable γ_j belongs to [0, 1], and τ is a global disorder parameter which defines the allowed displacement of the cylinders ($\tau \in]0,\ 1[$) as a proportion τ of the maximum permissible displacement $p = (d - a)$, with the properties at the limit: $\tau = 0 \Rightarrow$ no displacement of any cylinder, $\tau = 1 \Rightarrow$ maximal permissible displacement of the cylinders before contact, i.e. cylinder j possibly in contact with one of its neighbors. Thus, the perturbation parameter τ controls the level of disorder introduced in the array, while γ_j brings the randomness of this displacement for each cylinder j inside the permitted range. Of course τ is a parameter imposed by the user, whereas the γ_j are determined by a random generator computer routine. Fig. 3 gives a view of the results for three level of disorder: $\tau = 0.3$, $\tau = 0.5$ and $\tau = 0.9$ applied to the ordered reference array Fig. 2.

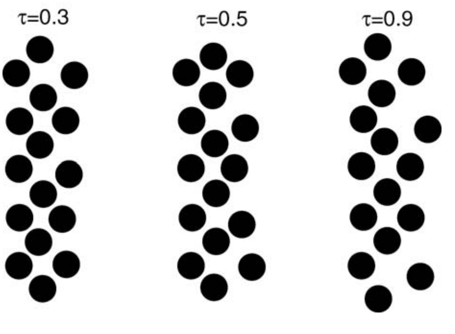

Fig. 3. Three level of disorder introduced in the ordered array Fig. 2.

3.1. Effect of disorder on near-trapped modes

Let us now turn to the first part of this study. The goal is to assess the sensitivity of the trapped mode waves with regard to a small deviation of the geometrical arrangement from the perfect disposition leading to this phenomenon. Evans and Porter (1997a,b) have shown that for a circular arrays of four cylinders like the arrangement visible in Fig. 1 (with $a/d = 0.8$), a near-trapped mode appears at wave frequency such that $\kappa d/\pi = 1.625293$ (or $\kappa a = 4.08482$) and wave incidence $\theta_{\mathrm{inc}} = 0$.

For the 16 cylinders, linear array constructed by aggregating this circular arrangement along the y axis (see Fig. 4), a trapped mode appears for the same wave frequency. As a consequence, very large forces are exerted upon the piles, except for cylinders numbered 7, 8, 9, 10 where total forces are less important due to some partial cancellation. Fig. 5 shows the force experienced by cylinder number 3 of the linear array (Fig. 2) and on cylinder 1 of the circular array (Fig. 1). For this critical value of κa, both curves present a very large and sharp peak, up to about 46 times the force on an identical isolated cylinder in the same wave field. Fig. 4 shows the free surface elevation magnification factor $|\phi|$ for this wavenumber for the linear array of regularly spaced piles. One can see that the amplitude of the free surface elevation between the cylinders is not the same for each cell of four cylinders, but that the figure is globally conserved and correspond to the scheme observed by Evans and Porter for the four piles circular array (see Fig. 10 in Evans and Porter, 1997a). For the cells located at the ends and for the middle cell, the free surface amplitude is similar to the one obtained for the circular array of four cylinders, the maximal wave amplitude is predicted to be about 150 times the incident wave amplitude. All around and behind the array, the wave amplitude keeps the same order of magnitude as the incident wave. In the area behind the array, the

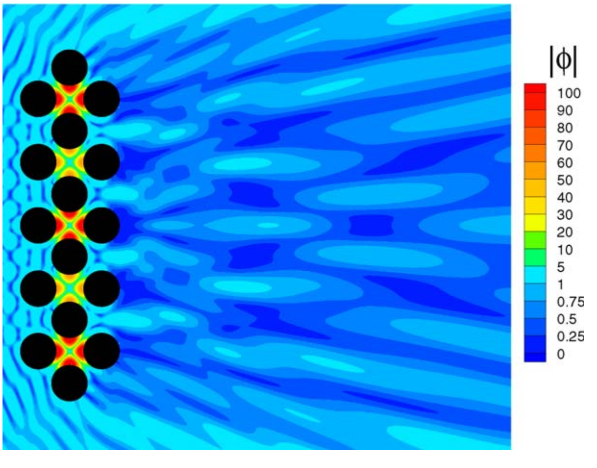

Fig. 4. Near-trapped mode for the ordered pile array at $\kappa d/\pi = 1.625293$.

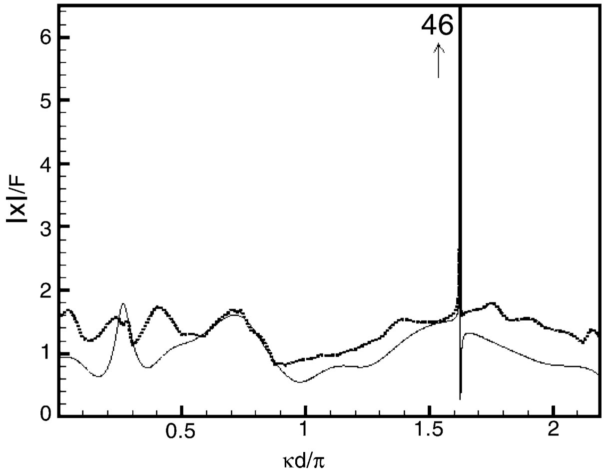

Fig. 5. Horizontal force on cylinder 1 of the circular array (Fig. 1) and (thick dotted line) on cylinder 3 of the large array (Fig. 2).

mean amplitude seems to be lower than unity, indicating a kind of sheltering effect which will be studied more closely in the next section.

Now, keeping the same wave frequency and the same basic array, we have introduced a certain level of disorder in the location of the cylinders, as explained in the previous section. For each value of the disorder parameter τ, 15 different pile groups corresponding to different sets of random parameter γ_j were tested. The

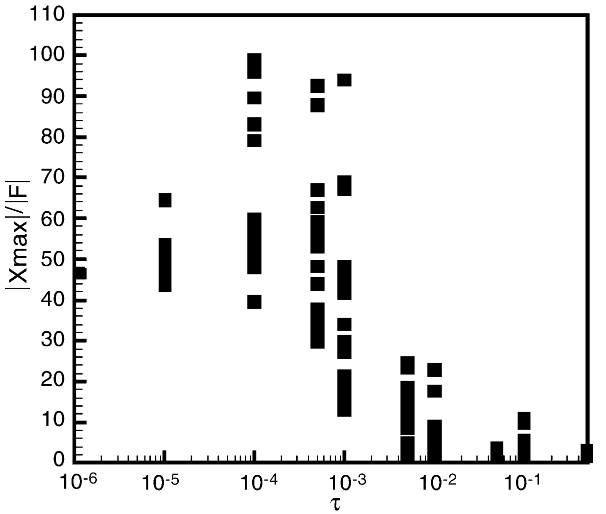

Fig. 6. Maximum resultant force versus disorder level parameter τ.

results are reported in Fig. 6, where the maximal force, reduced by the force on a single isolated cylinder, observed on the whole set of cylinders is represented as a function of the disorder levels (τ) for all the random parameter sets. As a first qualitative observation of these results, two different behavior of the maximum force exist, depending on whether τ being smaller or larger than 5×10^{-3}.

For low values ($\tau < 5 \times 10^{-3}$), the maximal force increases with the disorder level, from its reference value (46) corresponding to the trapped mode in the evenly spaced cylinder group, and it can be practically twice this value at the upper end of the range $10^{-4} < \tau < 5 \times 10^{-3}$. In this range, the displacement of the cylinders remains negligible; $\tau = 10^{-4}$ means a maximum displacement of 1 mm for a 10 m gap between cylinders. So it can be understood that the trapped mode phenomenon is still present, but that the maximum force can be even larger due to the disparition of some force cancellation by symmetry effects in the regular array.

Now for larger values of the disorder parameter $\tau \geq 5 \times 10^{-3}$, the maximal force is considerably and rapidly reduced as τ increases. It seems that the trapping mode phenomenon tends to disappears in the array with a disorder above this threshold value.

Let us now look to the sensitivity with regard to the frequency wave for a given disorder level. We start again from the evenly spaced reference array Fig. 2 as a basis, and we apply three disordering with a level $\tau = 0.1$, leading to the three unevenly spaced arrays presented in Fig. 7(a–c). Figs. 8–10 show the evolution of maximal forces with wave frequency for these three different unevenly spaced pile groups, the small narrows indicate the trapped mode frequency of the order array. These curves should be compared to the force on cylinder 3 of the regular array in Fig. 5 corresponding to the pure trapped mode. The frequency of this peak is marked by a vertical dashed line in Figs. 8–10. We can observe that, for the three displaced arrays, the trapped mode has been replaced by a moderate double peak extending over a wavenumber band of 0.1 width, which is far broader than the peak of the force in pure trapped mode, and that the amplitude of the peak is now

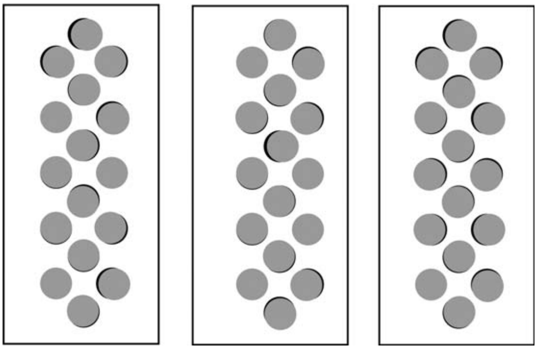

Fig. 7. Three different randomly displaced arrays (gray) (ordered array in black) with a disorder parameter $\tau = 0.1$.

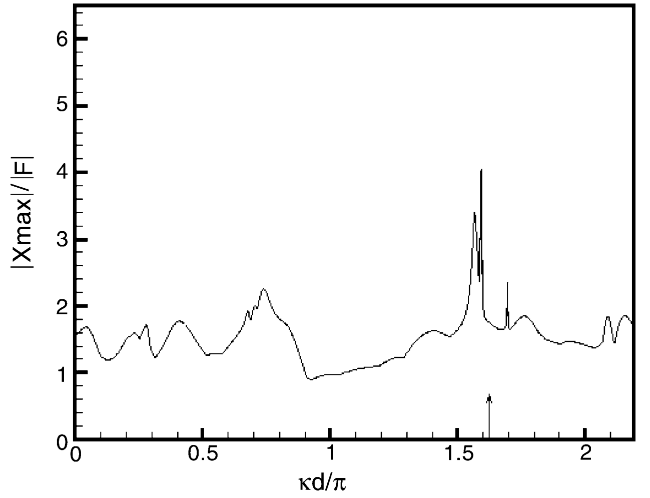

Fig. 8. Maximal force in unevenly ($\tau = 0.1$) spaced array shown in Fig. 7(a).

one order of magnitude lower. In Fig. 11, we have plotted the wave amplitude magnification factor for the original arrangement, and for the two last displaced arrangements denoted by (b) and (c) in Fig. 7. For such a disorder parameter $\tau = 0.1$, it is difficult to distinguish between the ordered (a) and displaced arrays (b) and (c); but the wave amplitude attenuation is immediately evident. From these figures, we can say that there is practically no more trapped mode in the unevenly spaced array at this disorder level.

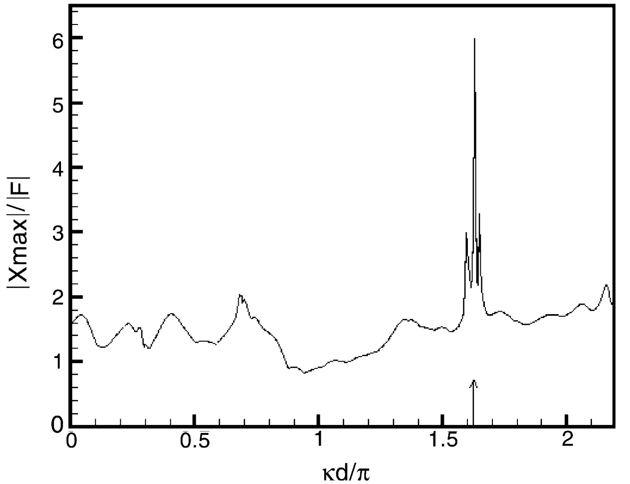

Fig. 9. Maximal force in unevenly ($\tau = 0.1$) spaced array shown in Fig. 7(b).

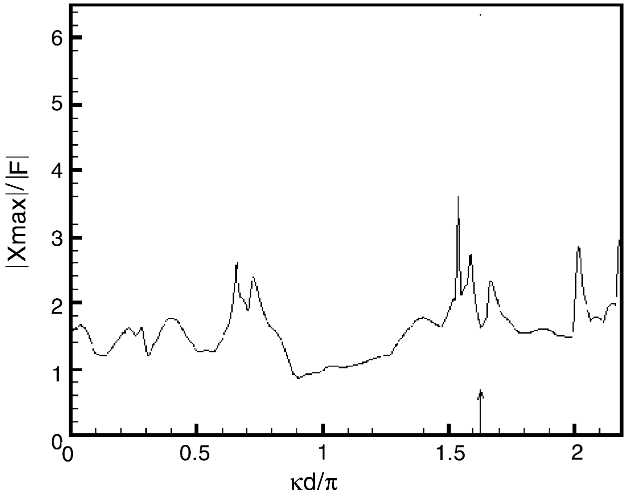

Fig. 10. Maximal force in unevenly ($\tau = 0.1$) spaced array shown in Fig. 7(c).

Another interesting feature of the wave pattern is revealed by this last figure Fig. 11. Comparing figures (b) and (c) to figure (a), it seems that introducing a small amount of disorder in the array reduce sensibly the mean wave amplitude behind it. From this observation, we have extended the initial scope of this study

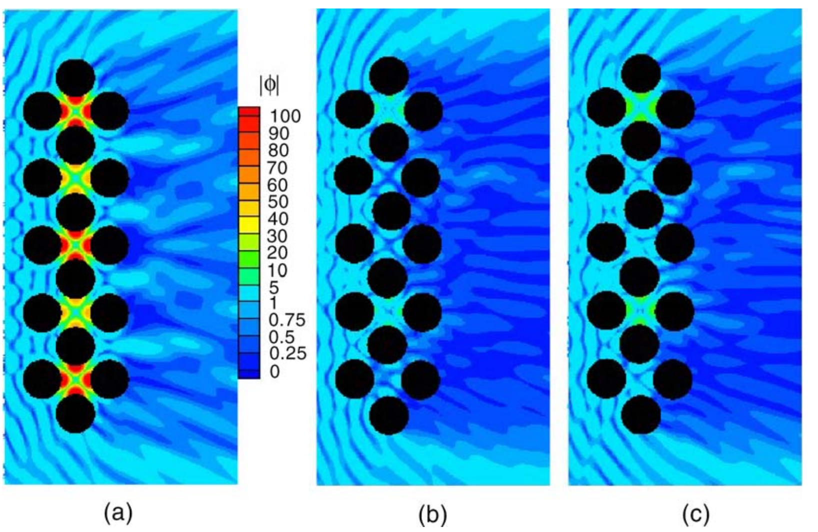

Fig. 11. Wave amplitude magnification at trapped mode wavenumber $\kappa d/\pi = 1.625293$. (a) Undisturbed reference array; (b) displaced array ($\tau = 0.1$) same as Fig. 7(b); (c) displaced array ($\tau = 0.1$) same as Fig. 7(c).

to assess the effect of disorder on the transmission of waves through such arrays of identical cylinders.

3.2. Effect of disorder on wave transmission through the array

Let us first define a reference array as previously by aggregating the basic four pile circular cell in a four rows array (38 cylinders) as depicted in Fig. 12. This array is indeed larger than the previous one, but still finite in length. The sheltered area will not therefore extend to infinity downstream, and we have defined a fixed triangular area (ABC) were a mean transmission is defined. Let ϕ_i be the incident velocity potential of unit amplitude propagating along the horizontal X axis, i.e. $\phi_i = \exp(ikx)$ and let ϕ_t be the total complex potential at any point M in the triangle ABC, still computed by Evan's method exposed in Section 2 above. ϕ_i being unitary, the modulus of ϕ_t can then be considered as a local transmission coefficient

$$|\phi_t(M)| = k_t(M)|\phi_i| = k_t(M) \tag{13}$$

Now let us define a mean transmission coefficient K_t by integrating k_t over the triangle ABC

$$K_t = \frac{1}{S_{ABC}} \int\int_{ABC} k_t(M)\mathrm{d}S \tag{14}$$

We shall now use this coefficient to quantify wave transmission across the pile array as we will progressively introduce some disorder in the cylinders arrangement. Different levels of disorder have been considered, and five of them are

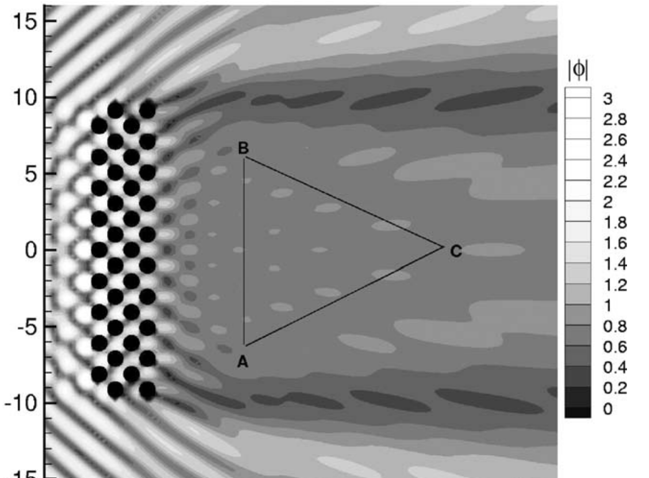

Fig. 12. Wave amplitude map around a 38 piles ordered array; triangular area (ABC) for wave transmission computation.

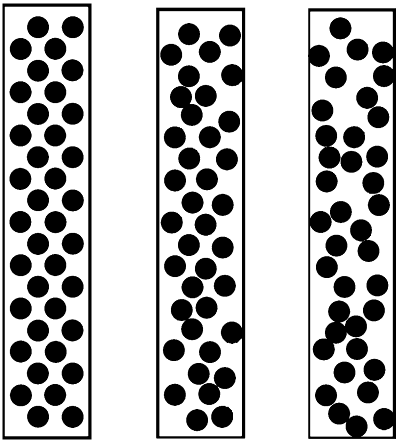

Fig. 13. Ordered reference array (left), and two disordered arrays: $\tau = 0.5$ (middle), $\tau = 0.9$ (right).

reported here: $\tau = 0$ (ordered array as a reference), then $\tau = 0.1$, 0.3, 0.5 and 0.9 (see examples Fig. 13). Again, the geometrical arrangement depends not only on τ but also on a set of random variables γ_j, each one being affected to cylinder j. The mean coefficient K_t therefore depends on each particular array arrangement. So we have proceeded to five different random choices of the γ_js for each level of disorder, in order to define an average coefficient $\langle K_t \rangle$ by $\langle K_t \rangle = (1/N)\sum_1^N K_t$, with here $N = 5$. This averaging is reported in Fig. 14 where five K_t curves and their average

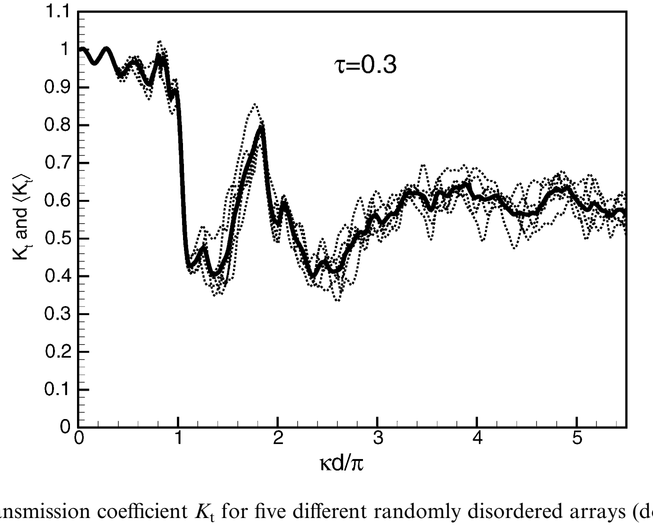

Fig. 14. Mean transmission coefficient K_t for five different randomly disordered arrays (dotted lines), and their average (bold black line) for a disorder level $\tau = 0.3$.

$\langle K_t \rangle$ are plotted as a function of the reduced wavenumber $\kappa d/\pi$, for a disorder parameter $\tau = 0.3$, to appreciate the typical standard deviation around this average curve.

Finally, this averaging method was applied to compute the transmission properties of the array over a wide frequency range, for four values of τ (plus $\tau = 0$) as mentioned above to assess the influence of disorder on the average transmission coefficient $\langle K_t \rangle$. All results are presented in a single figure (Fig. 15). For reduced wavenumber smaller than 1, it seems that the array is practically transparent to the waves whatever the level of disorder. The value $\kappa d/\pi = 1$ corresponds to a wavelength λ equal to $2d$ which is the typical spacing between cylinders centers; when $\kappa d/\pi < 1$, waves are longer than this spacing. Beyond this value, the coefficient falls abruptly to a value around 0.5. Then, as κ increases, the curve corresponding to the ordered array has a very oscillating behavior, reaching a value $\langle K_t \rangle = 0.1$ for $\kappa d/\pi = 1.7$ and falling to $\langle K_t \rangle = 0.1$ at $\kappa d/\pi = 2.7$. These large oscillations may be due to resonance phenomenon and are common in transmission coefficient curves for arrays of evenly spaced cylinders. After the critical value $\kappa d/\pi = 1$, we can observe on this example that the more disordered is the array, the less wavy is the curve. In the limit $\tau = 0.9$, the curve is practically flat about $\langle K_t \rangle = 0.6$, considering the noise introduced by the small number of cases Eq. (5) taken to compute the average. It seems that the same behavior holds when the disorder parameter decreases down to $\tau = 0.5$. After that, the wavy behavior of the regular array is progressively recovered.

This effect of disorder on wave transmission across the array could found some practical applications in coastal engineering, especially in the design of pile supported structures if one wants to limit, to enhance or to regularize wave

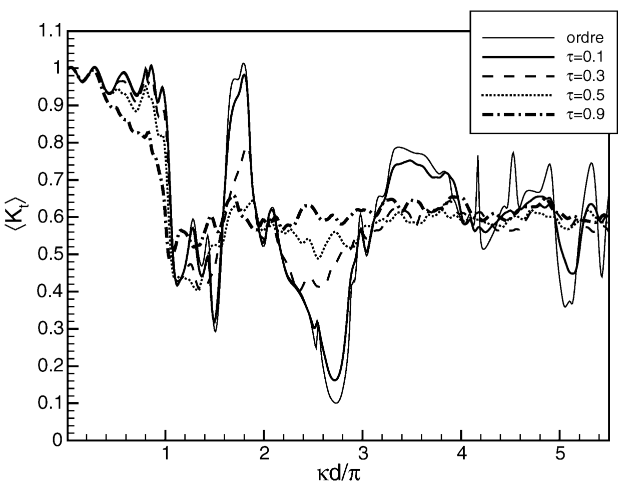

Fig. 15. Average transmission coefficient $\langle K_t \rangle$ versus reduced wavenumber $\kappa d/\pi$ for four different levels of disorder.

transmission depending on the frequency. In that case, $\tau = 0.5$ seems to be a reasonable threshold value for the disorder level.

4. Conclusion

Two aspects of the effect of uneven spacing on wave transmission through cylinders arrays have been investigated in the paper.

In the first part, we have shown on an example that a small level of disorder, beyond a certain threshold (5×10^{-3} of the maximum permissible displacement, in the present case) is sufficient to destroy the phenomenon of near-trapped modes, avoiding the very large forces associated to it.

In the second part, we have defined and computed an average transmission coefficient for a four pile rows group. This coefficient which is a wavy function of the wavenumber for evenly ordered arrays appears to be less and less oscillating as the level of disorder is increased.

These two properties of wave–array interaction, not well studied up to now, could have some interest for ocean engineers concerned with pile supported marine structures.

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