習題4-1

【基礎作業】

1. 將將習題 3-1 的內域問題改成外域問題，比較所得特性矩陣的差別

內域問題 (習題 3-1)

$$[T_i]\{u\} = [U_i]\{t\}$$

$$[M_i]\{u\} = [L_i]\{t\}$$

外域問題 (習題 4-1)

$$[T_e]\{u\} = [U_e]\{t\}$$

$$[M_e]\{u\} = [L_e]\{t\}$$

試比較 (1). U_i, U_e, (2). T_i, T_e, (3) L_i, L_e (4). M_i, M_e 的差別。

【進修作業】

1. 將 BECRAY.FOR 程式修改成外域問題，測試例如下

Governing equation:

$$\nabla^2 u(r,\theta) = 0, \ R < r < \infty, \ 0 < \theta < 2\pi$$

Boundary condition:

$$u(r,\theta) = f(\theta), \ for \ r = R$$

Please derive the Poisson formula for exterior domain.

$$u(\rho,\theta) = \frac{1}{2\pi} \int_0^{2\pi} \frac{\rho^2 - R^2}{R^2 + \rho^2 - 2R\rho \ cos(\theta - \theta')} f(\theta')d\theta'$$

Solve the above exterior problem either analytically or numerically for the following B.C.

$$f(\theta) = \pm 1.0, \ + \ for \ 0 < \theta < \pi, \ - \ for \ \pi < \theta < 2\pi$$

where the radius is $R = 1$.

Plot the potential and potential gradient along the three angles 30, 60, 90 degrees from $\rho = 1$ to $\rho = 5$. Also, plot the normal flux on the circular boundary.