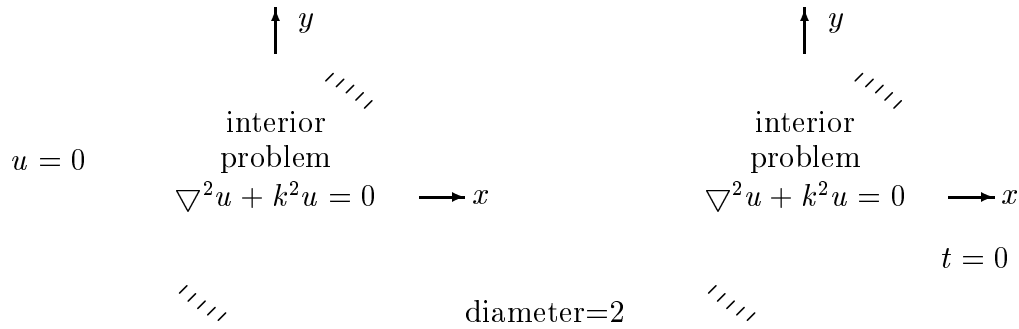


程式14-3 Exact solution



1. Problem statement:
Governing equation:

$$(\nabla^2 + k^2)u(r, \theta) = 0, (r, \theta) \text{ in } D$$

Boundary conditions:

$$u(a, \theta) = 0, (a, \theta) \text{ on the boundaries}$$

2. Table 1 Exact solution for membrane without stringers ($a = 1, u = 0$ on B)

No. (n)	eigenvalues (k_n)	eigen equation	eigenmode $u_n(r, \theta)$	multiplicity
1	2.4048(2.4070)	$J_0(ka) = 0$	$J_0(2.4048r)$	1
2,3	3.8317(3.8342)	$J_1(ka) = 0$	$J_1(3.8317r)e^{\pm i\theta}$	2
4,5	5.1356(5.1388)	$J_2(ka) = 0$	$J_2(5.1356r)e^{\pm i2\theta}$	2
6	5.5201(5.5223)	$J_0(ka) = 0$	$J_0(5.5201r)$	1
7,8	6.3802(6.3837)	$J_3(ka) = 0$	$J_3(6.3802r)e^{\pm i3\theta}$	2
9,10	7.0156(7.0189)	$J_1(ka) = 0$	$J_1(7.0156r)e^{\pm i\theta}$	2
11,12	7.5883(7.5920)	$J_4(ka) = 0$	$J_4(7.5883r)e^{\pm i4\theta}$	2
13,14	8.4172(8.4219)	$J_2(ka) = 0$	$J_2(8.4172r)e^{\pm i2\theta}$	2
15	8.6537(8.6591)	$J_0(ka) = 0$	$J_0(8.6537r)$	1
16,17	8.7715(8.7742)	$J_5(ka) = 0$	$J_5(8.7715r)e^{\pm i5\theta}$	2
18,19	9.7610(9.7669)	$J_3(ka) = 0$	$J_3(9.7610r)e^{\pm i3\theta}$	2
20,21	9.9361(9.9404)	$J_6(ka) = 0$	$J_6(9.9361r)e^{\pm i6\theta}$	2

3. Table 2 Exact solution for membrane without stringers ($a = 1, t = 0$ on B)

No. (n)	eigenvalues (k_n)	eigen equation	eigenmode $u_n(r, \theta)$	multiplicity
1	0.0000(0.0000)	$J'_0(ka) = 0$	$J_0(0.0000r)$	1
2,3	1.8412(1.8436)	$J'_1(ka) = 0$	$J_1(1.8412r)e^{\pm i\theta}$	2
4,5	3.0542(3.0586)	$J'_2(ka) = 0$	$J_2(3.0542r)e^{\pm i2\theta}$	2
6	3.8317(3.8364)	$J'_0(ka) = 0$	$J_0(3.8317r)$	1
7,8	4.2012(4.2073)	$J'_3(ka) = 0$	$J_3(4.2012r)e^{\pm i3\theta}$	2
9,10	5.3176(5.3242)	$J'_4(ka) = 0$	$J_4(5.3176r)e^{\pm i4\theta}$	2
11,12	5.3314(5.3309)	$J'_1(ka) = 0$	$J_1(5.3314r)e^{\pm i\theta}$	2
13,14	6.4156(6.4260)	$J'_5(ka) = 0$	$J_5(6.4156r)e^{\pm i5\theta}$	2
15,16	6.7061(6.7137)	$J'_2(ka) = 0$	$J_2(6.7061r)e^{\pm i2\theta}$	2
17	7.0156(7.0230)	$J'_0(ka) = 0$	$J_0(7.0156r)$	1
18,19	7.5883(7.5124)	$J'_6(ka) = 0$	$J_6(7.0356r)e^{\pm i7\theta}$	2
20,21	8.0156(8.0247)	$J'_3(ka) = 0$	$J_3(8.0156r)e^{\pm i3\theta}$	2