程式 101 Biharmonic problems with holes

Governing equation： $\nabla^4 u(x) = 0,\ x \in \Omega$

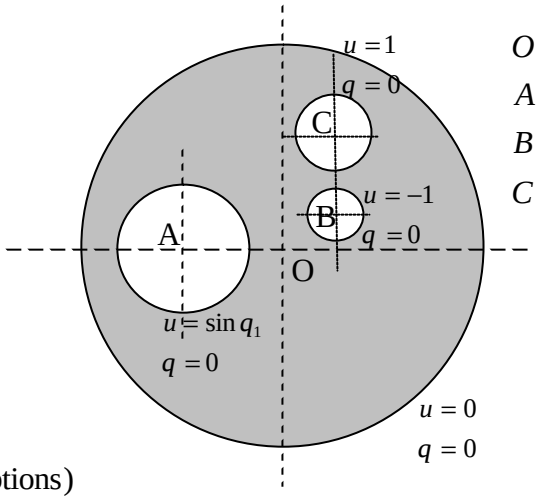
B.C.：Fixed $u(x)\big|_{x\in B} = 0$， $q(x)\big|_{x\in B} = 0$

Simply-supported $u(x)\big|_{x\in B} = 0$， $m(x)\big|_{x\in B} = 0$

Free $m(x)\big|_{x\in B} = 0$， $v(x)\big|_{x\in B} = 0$

1. Degenerate kernel：

$$U(s,x) = r^2 \ln r = \begin{cases} U^I(s,x) = r^2(1+\ln R) + R^2 \ln R - Rr(1+2\ln R)\cos(q-f) - \sum_{m=1}^{\infty} \frac{1}{m(m+1)} \frac{r^{m+2}}{R^m} \cos[m(q-f)] + \sum_{m=2}^{\infty} \frac{1}{m(m-1)} \frac{r^m}{R^{m-2}} \cos[m(q-f)],\ R > r \\ U^E(s,x) = R^2(1+\ln r) + r^2 \ln r - rR(1+2\ln r)\cos(q-f) - \sum_{m=1}^{\infty} \frac{1}{m(m+1)} \frac{R^{m+2}}{r^m} \cos[m(q-f)] + \sum_{m=2}^{\infty} \frac{1}{m(m-1)} \frac{R^m}{r^{m-2}} \cos[m(q-f)],\ r > R \end{cases}$$



$O = (0,0),\ R_O = 20$
$A = (-14,0),\ R_A = 5$
$B = (5,3),\ R_B = 2$
$C = (5,10),\ R_C = 4$

2. Fourier series expansion

$$u(s) = a_0 + \sum_{n=1}^{\infty} a_n \cos nq + b_n \sin nq$$

$$q(s) = c_0 + \sum_{n=1}^{\infty} c_n \cos nq + d_n \sin nq$$

$$m(s) = g_0 + \sum_{n=1}^{\infty} g_n \cos nq + h_n \sin nq$$

$$v(s) = p_0 + \sum_{n=1}^{\infty} p_n \cos nq + q_n \sin nq$$

3. Null-field integral equations (C_2^4 options)

$$0 = \int_B \{-U(s,x)v(s) + \Theta(s,x)m(s) - M(s,x)q(s) + V(s,x)u(s)\} dB(s),\ x \in \Omega^e$$

$$0 = \int_B \{-U_q(s,x)v(s) + \Theta_q(s,x)m(s) - M_q(s,x)q(s) + V_q(s,x)u(s)\} dB(s),\ x \in \Omega^e$$

$$0 = \int_B \{-U_m(s,x)v(s) + \Theta_m(s,x)m(s) - M_m(s,x)q(s) + V_m(s,x)u(s)\} dB(s),\ x \in \Omega^e$$

$$0 = \int_B \{-U_v(s,x)v(s) + \Theta_v(s,x)m(s) - M_v(s,x)q(s) + V_v(s,x)u(s)\} dB(s),\ x \in \Omega^e$$

4. Boundary integral equations for the interior potential

References：

1. W. C. Shen, C. F. Lee, and J. T. Chen, A Study on Half-Plane Laplace Problems with a Circular Hole, **第七屆結構工程研討會**, **大溪**, 2004.
2. W. C. Shen, J. T. Chen, and C. F. Lee, A Study on Laplace of Infinite Plane with Multiple Circular Holes, International Conference on Computational Methods, Singapore, 2004.
3. M. D. Bird and C. R. Steele, Separated Solution Procedure for Bending of Circular Plates with Circular Holes, Applied Mechanics Reviews, No. 11, Part 2, pp. s27-s35, 1991.