程式27 Half-plane problem

Laplace problem : ($u = 0$ on half plane)

$\underline{u = 0 \text{ on half plane}}$

D^e

Laplace problem : ($t = 0$ on half surface)

1. **Problem statement:**

   Governing equation:

   $$\nabla^2 u(r, \theta) = 0, (r, \theta) \ in \ D^e$$

   Boundary condition:

   $$u(1, \theta) = 1, \ 0 < \theta < 2\pi$$

2. Output : field distribution

3. Analytical solution: (by fractional linear transformation)

   $$u(x, y) = \frac{1}{ln[(h + c)/a]} ln \frac{\sqrt{(x^2 - c^2 + y^2)^2 + 4c^2 y^2}}{(x - c)^2 + y^2}, \ (x, y) \in D^e$$

   where

   $$c = \sqrt{(h^2 - a^2)}$$

4. Analytical solution: (by image method and degenerate kernel)

   $$\frac{\partial u}{\partial n} \ \text{distribution along the half and circular boundary}$$

References

[1] Lebedev, N. N., Skalskaya I. P. and Ulfyans, Y. S., Worked problems in applied mathematics, translated by Silverman, R. A., pp.37 and 213, Dover, New York, 1965.

[2] 李慶鋒，半平面與多連通拉普拉斯問題研究，海洋大學河海工程研究所碩士論文，2001。