

程式28 Half-plane problem

Laplace problem : ($u = 0$ on half plane)

$$\frac{u = 0 \text{ on half plane}}{D^e}$$

Laplace problem : ($t = 0$ on half surface)

1. Problem statement:

Governing equation:

$$\nabla^2 u(r, \theta) = 0, (r, \theta) \text{ in } D^e$$

Boundary condition:

$$t(1, \theta) = 1, 0 < \theta < 2\pi$$

2. Output : field distribution

3. Analytical solution: (by fractional linear transformation)

$$u(\alpha, \beta) = \frac{Q}{k\pi \sinh \alpha_0} \left[\frac{\alpha}{2} + \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \frac{e^{-n\alpha_0}}{\cosh(n\alpha_0)} \sinh n\alpha \cos n\beta \right],$$

where

$$\cosh \alpha_0 = \frac{b}{a}, k \text{ is the thermal conductivity, } Q \text{ is the heat current.}$$

4. Analytical solution: (by image method and degenerate kernel)

u distribution along the half and circular boundary

References

- [1] Lebedev, N. N., Skalskaya I. P. and Ulfyans, Y. S., Worked problems in applied mathematics, translated by Silverman, R. A., pp.37 and 213, Dover, New York, 1965.
- [2] 李慶鋒，半平面與多連通拉普拉斯問題研究，海洋大學河海工程研究所碩士論文，2001。