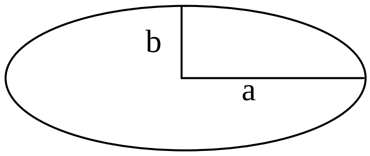
程式 85 Degenerate scale for an elliptical domain

$x = r_e (a\cos f, b\sin f)$

$s = R_e (a\cos q, b\sin q)$



$\frac{b}{a} = r$

1. Degenerate kernel $U(s,x) = \Sigma X_i(x) S_i(s)$
2. $Tu_n(q) = l_n u_n(q)$

$$l_n = \begin{cases} -\frac{1}{2}(\frac{1-r}{1+r})^n \ , n > 0 \\ \frac{-1}{2} \ , n = 0 \\ -\frac{1}{2}(\frac{1-r}{1+r})^{-n} \ , n < 0 \end{cases} \qquad u_n(q) = \begin{cases} \cos(nq) \ , n > 0 \\ \frac{1}{\sqrt{2}} \ , n = 0 \\ \sin(nq) \ , n < 0 \end{cases}$$

3. $Uw_n(q) = m_n J(q) w_n(q)$

$$m_n = \begin{cases} \frac{1}{2n}\left[1 + (\frac{1-r}{1+r})^n\right], n > 0 \\ -\log(\frac{a+b}{2}) \ , n = 0 \\ -\frac{1}{2n}\left[1 - (\frac{1-r}{1+r})^{-n}\right], n < 0 \end{cases}$$

where $J(q) = \sqrt{a^2 \sin^2 q + b^2 \cos^2 q}$, $w_n(q) = J^{-1}(q) u_n(q)$

4. $Mu_n(q) = h_n J^{-1}(q) u_n(q)$

$$h_n = \begin{cases} 2n\left[\frac{1}{2} - (\frac{1-r}{1+r})^n\right], n > 0 \\ 0 \ , n = 0 \\ -2n\left[\frac{1}{2} + (\frac{1-r}{1+r})^{-n}\right], n < 0 \end{cases}$$

【References】

1. G.J. Rodin and O. Steinbach, Boundary element preconditioner for problems defined on slender domains, SIAM J. Sci. Comp., Vol. 24, No.4, pp.1450-1464, 2003.