程式 88 Updating technique for beam

In the BEM course, we have four equations

$u(s)=\left[-U(x,s)u'''(x)+\Theta(x,s)u''(x)-M(x,s)u'(x)+V(x,s)u(x)\right]\Big\vert_{x=0}^{x=1}$	(a)
$u'(s)=\left[-U_q(x,s)u'''(x)+\Theta_q(x,s)u''(x)-M_q(x,s)u'(x)+V_q(x,s)u(x)\right]\Big\vert_{x=0}^{x=1}$	(b)
$u''(s)=\left[-U_m(x,s)u'''(x)+\Theta_m(x,s)u''(x)-M_m(x,s)u'(x)+V_m(x,s)u(x)\right]\Big\vert_{x=0}^{x=1}$	(c)
$u'''(s)=\left[-U_v(x,s)u'''(x)+\Theta_v(x,s)u''(x)-M_v(x,s)u'(x)+V_v(x,s)u(x)\right]\Big\vert_{x=0}^{x=1}$	(d)

By using any two, we have

$A_1\underset{\sim}{u}=B_1\underset{\sim}{p}$	(1)	$\underset{\sim}{u}=\begin{Bmatrix} u(0)\\ q(0)\\ u(1)\\ q(1)\end{Bmatrix}$	$\underset{\sim}{p}=\begin{Bmatrix} m(0)\\ v(0)\\ m(1)\\ v(1)\end{Bmatrix}$
$A_2\underset{\sim}{u}=B_2\underset{\sim}{p}$	(2)		
$A_3\underset{\sim}{u}=B_3\underset{\sim}{p}$	(3)		
$A_4\underset{\sim}{u}=B_4\underset{\sim}{p}$	(4)		
$A_5\underset{\sim}{u}=B_5\underset{\sim}{p}$	(5)		
$A_6\underset{\sim}{u}=B_6\underset{\sim}{p}$	(6)		

SVD updating document $[A_i \mid B_i]$ $(i=1,2,\cdots\cdots 6)$ $\Longrightarrow$ 6 cases

SVD updating term $\begin{bmatrix} A_i\\ \hline A_j\end{bmatrix}$ $(i<j,\quad i=1,2,\cdots\cdots 5)$ $\Longrightarrow$ $C_2^6=15$ cases

Case1：$\underset{\sim}{u}$ is specified $u=\bar{u}$, $\underset{\sim}{p}$ is unknown (Dirichlet problem)

$\Longrightarrow$ $\underset{\sim}{f}^T[A_i \mid B_i]=\underset{\sim}{0}$ $\Longrightarrow$ $\underset{\sim}{f}$

$\underset{\sim}{y}^B=\underset{\sim}{p}$ and $\underset{\sim}{u}=\underset{\sim}{\bar{u}}$ $\Longrightarrow$ 代入（a）$\Longrightarrow$ Null field solution

Case2：$\underset{\sim}{u}$ is unknown, $\underset{\sim}{p}=\langle 0,0,0,0\rangle^T$ (Free-free problem)

$\Longrightarrow$ $\begin{bmatrix} A_i\\ \hline A_j\end{bmatrix}\underset{\sim}{y}=0$

$\underset{\sim}{y}^A=\underset{\sim}{u}$ and $\underset{\sim}{p}=\langle 0,0,0,0\rangle^T$ $\Longrightarrow$ 代入（a）$\Longrightarrow$ Rigid body mode

$$[A_i] \quad \Downarrow$$

$$\begin{bmatrix} \cdots & \underset{\sim}{f}^{A} & \cdots \end{bmatrix} \begin{bmatrix} \ddots & & \\ & 0 & \\ & & \ddots \end{bmatrix} \begin{bmatrix} \cdots & \underset{\sim}{y}_{i} & \cdots \end{bmatrix}$$

$$[B_i] \quad \Downarrow$$

$$\begin{bmatrix} \cdots & \underset{\sim}{f}^{B} & \cdots \end{bmatrix} \begin{bmatrix} \ddots & & \\ & 0 & \\ & & \ddots \end{bmatrix} \begin{bmatrix} \cdots & \underset{\sim}{y}_{j} & \cdots \end{bmatrix}$$

$\underset{\sim}{f}^{A} = \underset{\sim}{f}^{B}$ (spurious mode)

$$[A_j] \quad \Downarrow$$

$$\begin{bmatrix} \cdots & \underset{\sim}{f}^{B} & \cdots \end{bmatrix} \begin{bmatrix} \ddots & & \\ & 0 & \\ & & \ddots \end{bmatrix} \begin{bmatrix} \cdots & \underset{\sim}{y}_{j} & \cdots \end{bmatrix}^{T}$$

$\underset{\sim}{y}_{i} = \underset{\sim}{y}_{j}$ (rigid body mode)