班級：____________學號：_____________姓名：_____________

國立台灣海洋大學河海工程研究所 BEM 作業(2003)

1. Please use on-line library system of NTOU Library, download one paper (2003) using PDF file related to integral equations or boundary element method.
2. Solve the fundamental solution using any method you can

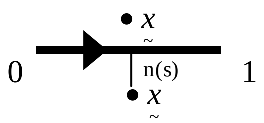
$$\frac{d^2U(x,s)}{dx^2}=d(x-s)$$

3. Solve the fundamental solution using Fourier transform with single pole ($q=0$) and double poles ($q\neq 0$).

$$\frac{d^2U(x,s)}{dx^2}-q^2U(x,s)=d(x-s)$$

4. Derive the null-field integral equations ($s=0^-$ and 1^+) for UT and LM equations.
5. Derive the beam stiffness using any two equations of dual formulation.
6. Find the displacement field of a cantilever beam subject to the end shear (1N).
7. Derive the 3-D fundamental solution of $\nabla_x^2 U(x,s)=d(x-s)$.
8. Derive the $M(s,x)$ for 2-D Laplace equation.
9. Derive the dual BIEs by using the bump contour approach where the singularity is outside the domain. Find the errors of pp.103-106 in Chen and Hong's BEM textbook.
10. Calculate the four integrals $\int_0^1 U(s,x)t(s)ds$, $\int_0^1 T(s,x)u(s)ds$, $\int_0^1 L(s,x)t(s)ds$ and

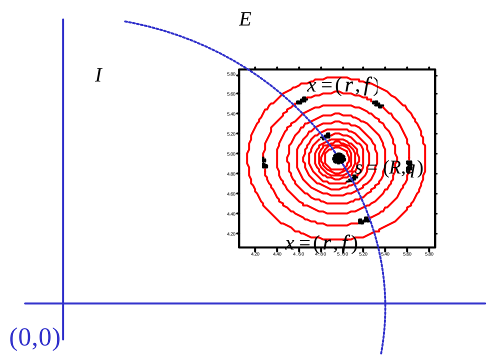
$\int_0^1 M(s,x)u(s)ds$ by using symbolic software, where $\underset{\sim}{x}=(0.5,e)$, $\underset{\sim}{s}=(s,0)$, $n(s)=(0,-1)$ and $n(x)=(0,-1)$.



Please find the limiting values for $e\to 0^+$ and $e\to 0^-$.

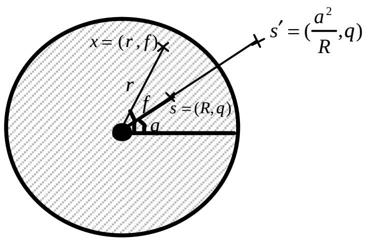
11. Please plot the kernel function $\ln r$ in terms of degenerate kernel as shown below.

$$\ln r=\begin{cases}\ln R-\sum_{m=1}^{\infty}\frac{1}{m}(\frac{r}{R})^m(\cos m(q-f)), & R>r\\ \ln r-\sum_{m=1}^{\infty}\frac{1}{m}(\frac{R}{r})^m(\cos m(q-f)), & R<r\end{cases}$$



12. (1) Plot Green's function $U_p(x,s) = \ln|x-s| - \ln|x-s'| + \ln a - \ln R$.

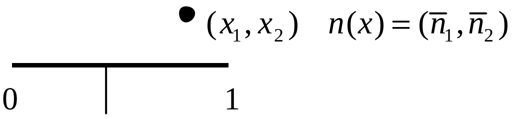
(2) Derive the Poisson Integral formula.



$$U(r,f) = \frac{1}{2p}\int_0^{2p} \frac{a^2 - r^2}{a^2 + r^2 - 2ar\cos(f-q)} f(q)dq$$

(3) Plot the Green's function using series form (degenerate kernel).

13. Regular integral (1)Closed-form (2)Numerical integration



Calculate (1) $\int_B U(s,x)dB(s)$ (2) $\int_B T(s,x)dB(s)$ (3) $\int_B L(s,x)dB(s)$

(4) $\int_B M(s,x)dB(s)$, where $(x_1,x_2) = (1,0.5)$, $n(s) = (0,-1)$ and $n(x) = (1,0)$

Construct the Table for Gaussian points and weightings. (N=1,2,3,4,5)

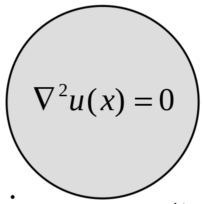
14.1 Determine the following integral by Gaussian quadrature $\int_{-1}^{1} \frac{\cos(x)\sqrt{1-x^3}}{x}dx$

(1) Direct method.

(2) Folding technique.

(3) Addition and subtraction method.

14.2 In the course, we used null-field integral equation



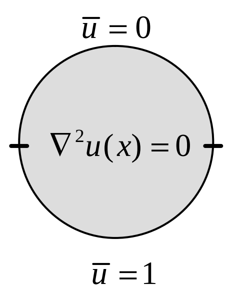
$$0 = \int_B T^E(s,x)u(s)dB(s) - \int_B U^E(s,x)t(s)dB(s)$$ to obtain $t(1,q) = \cos q$.

Substituting the degenerate kernels $U^I(s,x)$, $T^I(s,x)$, $u(1,q) = \cos q$, and

$t(1,q) = \cos q$ to $2pu(x) = \int_B T^I(s,x)u(s)dB(s) - \int_B U^I(s,x)t(s)dB(s)$ to obtain the

exact solution $u(r,f)$, $0 \le r \le 1$, $0 < f < 2p$, where

$$U(s,x) = \ln r = \begin{cases} U^I = \ln R - \sum_{m=1}^{\infty} \frac{1}{m}(\frac{r}{R})^m (\cos m(q-f)), & R > r \\ U^E = \ln r - \sum_{m=1}^{\infty} \frac{1}{m}(\frac{R}{r})^m (\cos m(q-f)), & R < r \end{cases}$$



15. Solve $\nabla^2 u = 0$ subject to $u(1,f) = 0$, $0 < f < p$ and $u(1,f) = 1$, $p < f < 2p$ by using (1) exact solution (complex variable) $u(x,y) = \frac{1}{p}\tan^{-1}(\frac{1-x^2-y^2}{2y})$ (2) Poisson integral formula (Gaussian quadrature) (3) series solution (4) BEM (BEPO2D program)