程式 44 Numerical instability

1. Given the linear algebraic system,

$$\begin{bmatrix} 1 & 0 \\ 0 & \alpha\varepsilon \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ \beta\varepsilon \end{Bmatrix}, \quad \alpha, \beta, \varepsilon \in R,$$

① Solve x_1 and x_2 analytically.

② Solve x_1 and x_2 numerically by setting $\varepsilon \to 0$.

③ Solve x_1 and x_2 in the transformed system

$$\left[A + c\tilde{b}\tilde{b}^T\right]\tilde{x} = \tilde{b}$$

where c is an arbitrary constant.

④ Solve

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha\varepsilon + p & -p \\ 0 & -p & p \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 1 \\ \beta\varepsilon \\ 0 \end{Bmatrix},$$

where p is an arbitrary constant.

⑤Slove x_1 and x_2 by using pseudo-inverse when $\varepsilon \to 0$.